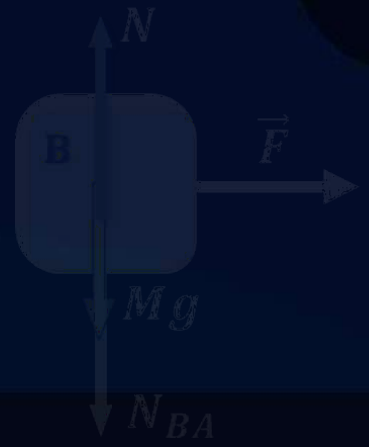


Welcome to

Laws of Motion

Laws of Motion




Mechanics
Kinematics
Dynamics

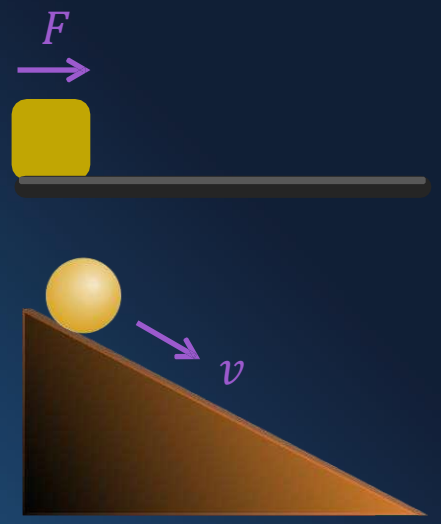
Introduction

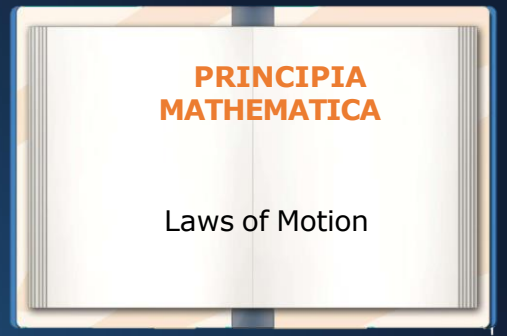


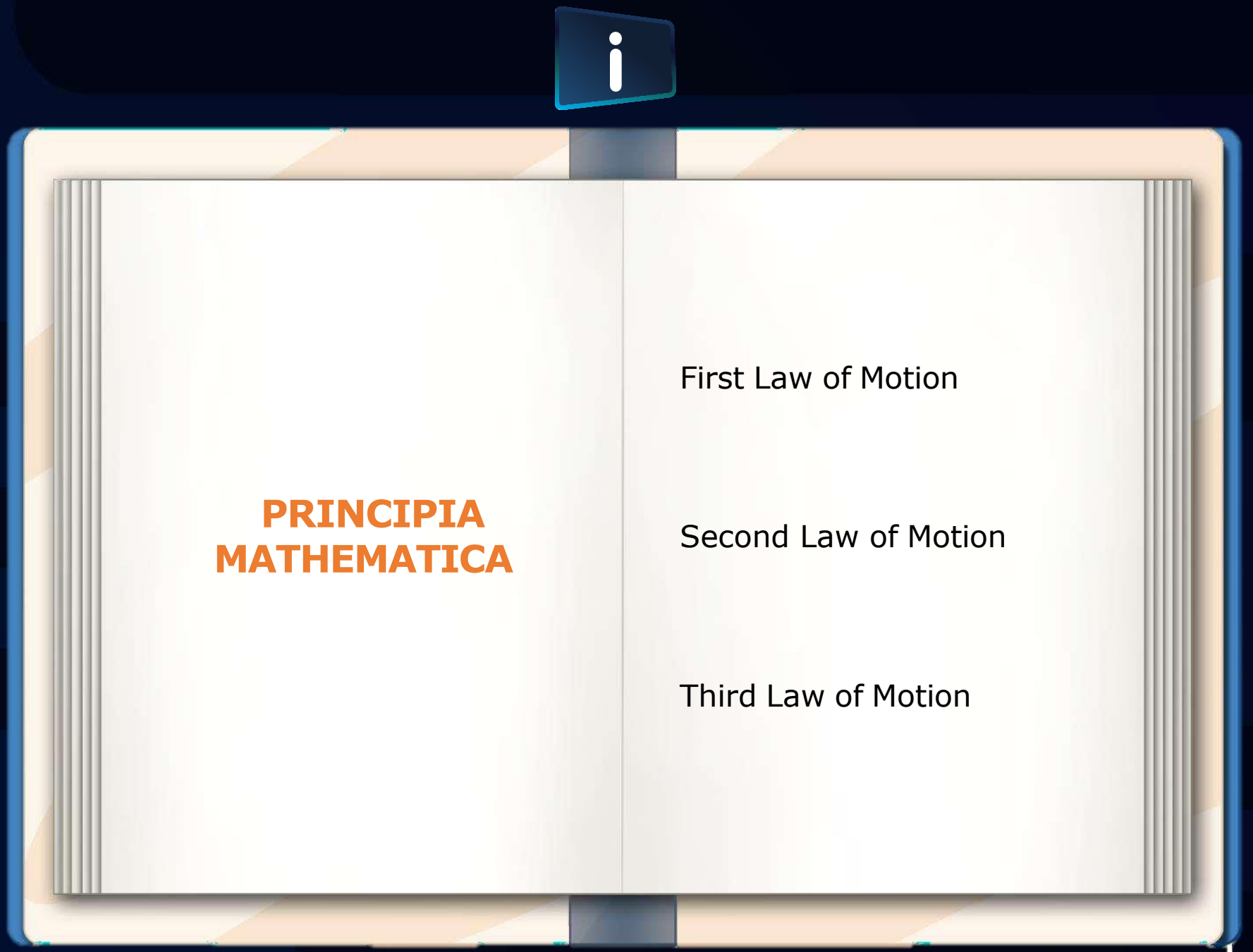



All objects have a natural place in the universe.
The natural state of a body is rest.

In the absence of force, a moving object will continue moving.







PRINCIPIA MATHEMATICA

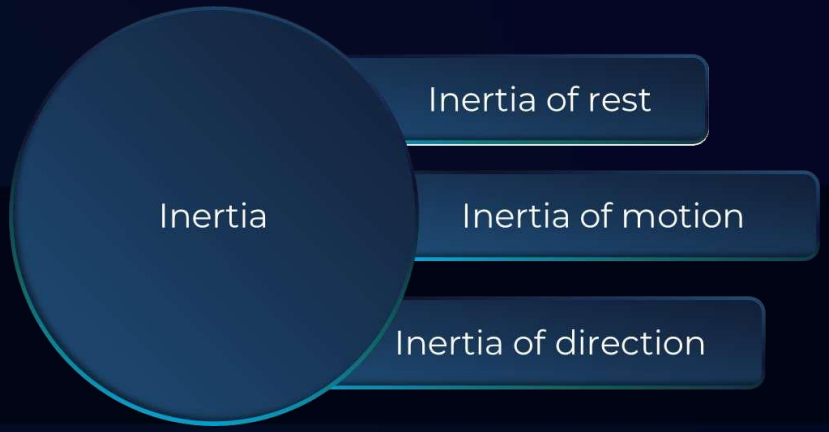
First Law of Motion

Second Law of Motion

Third Law of Motion

Inertia

- Inertia – The resistance of a particle to change its state of rest or uniform motion along a straight line.



Types of Inertia



Inertia of rest :
Initial state of the body is rest.

The body resists motion.



Inertia of motion :
Initial state of the body is motion.

The body resists coming to a stop.



Inertia of direction :
The body resists change in direction.

Inertia





- Inertia is a property of the mass of the particle.

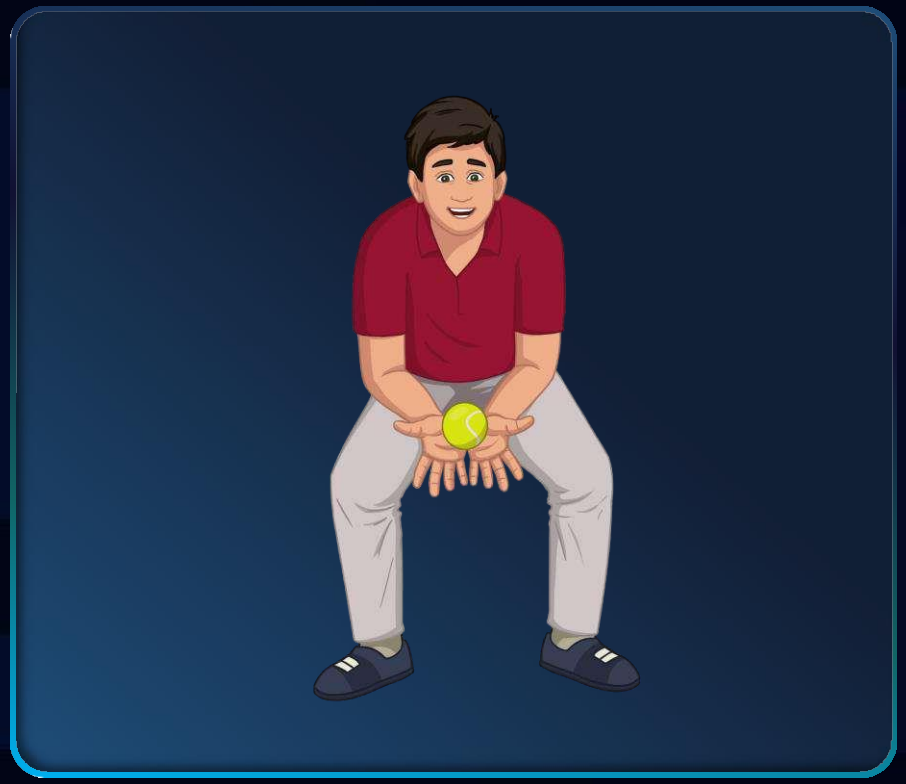
  Inertia $\propto$ mass

- For rotational motion, it not only depends on mass but also on the distance from the axis of rotation, this parameter is called the moment of inertia.

First Law :Law of Inertia

- Every object will remain at rest or uniform motion in a straight line until and unless compelled to change its state by action of net external force.





Force

- Force - A push or pull that changes or tends to change the state of motion.
- Force – can change direction of motion of a body or speed of a body or both or even shape of a body.
- Force is a vector quantity, it has both magnitude and direction.
- Symbol - F and SI unit - N (Newton)



Net Force

- The vector sum of all the external forces acting on a system represents the net force on the that system.

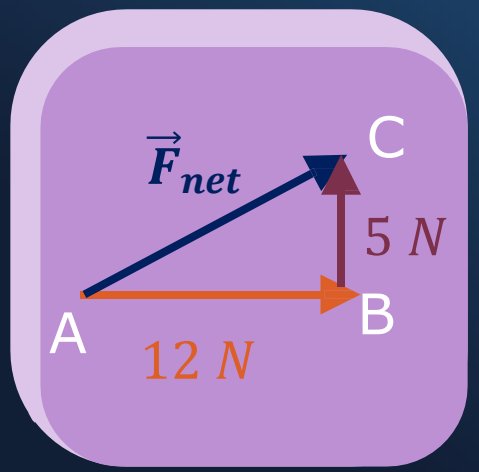
$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ \ldots$$



- If $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = 0$, then the object will continue to do what it has been doing.

Find the net force acting on the system. If the body was at rest before the application of force, what will be its state because of net force acting on it?

Solution :



$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$

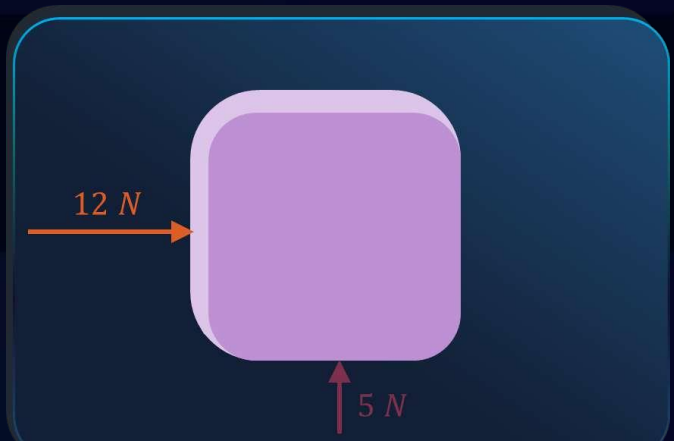
$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 12^2 + 5^2$$

$$AC^2 = 169$$

$$AC = 13$$

$$\vec{F}_{net} = 13\ N$$

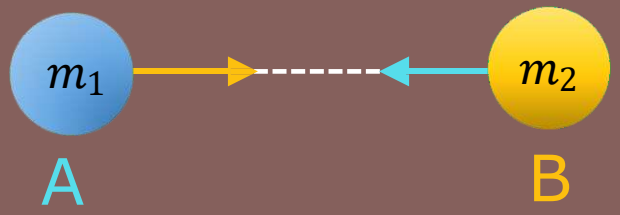


There is net force acting on the object hence, it starts to move in the direction of net force.

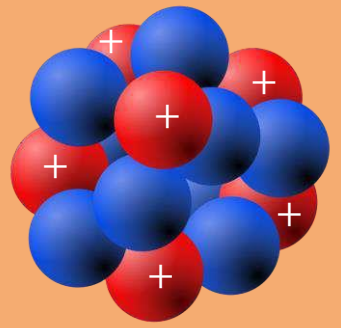
Fundamental Forces in Nature

Fundamental Forces

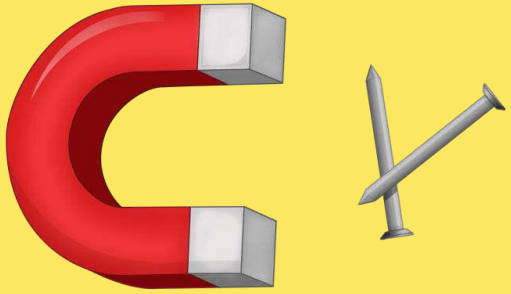
Gravitational



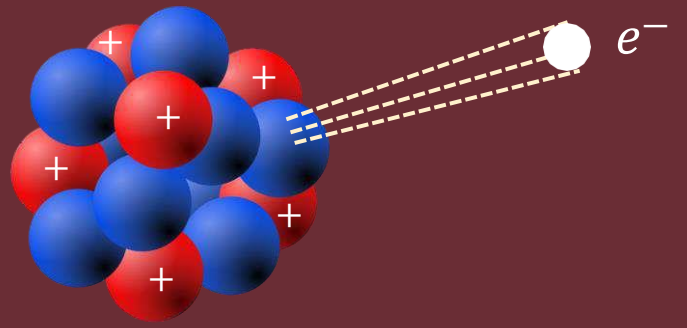
Strong Nuclear



Electromagnetic

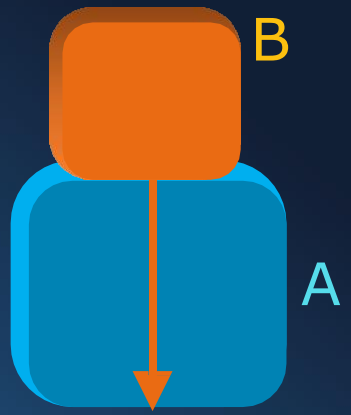


Weak Nuclear



Forces

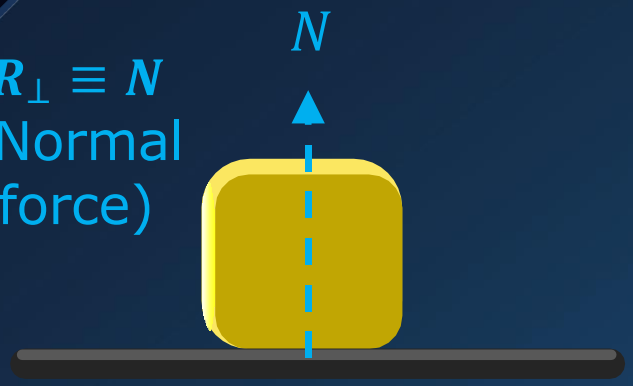
Contact Forces



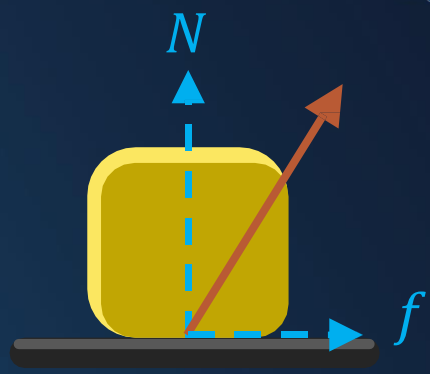
Non-Contact Forces

Normal Reaction & Friction

$R_{\perp} \equiv N$
(Normal force)



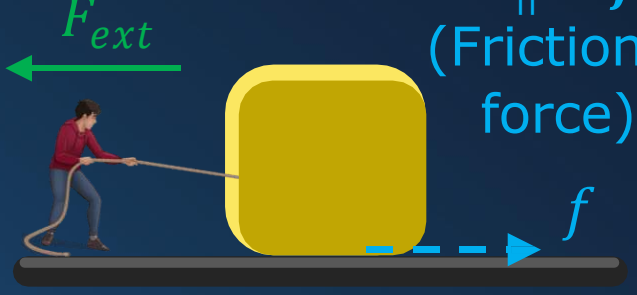
Perpendicular component



The resultant is called contact force.

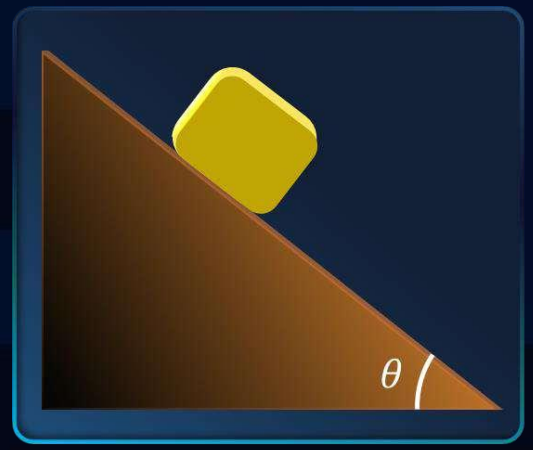
Contact force is electrodynamic in nature.

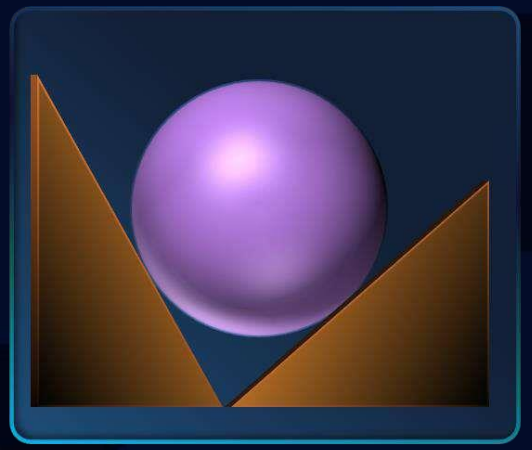
$R_{||} \equiv f$
(Frictional force)

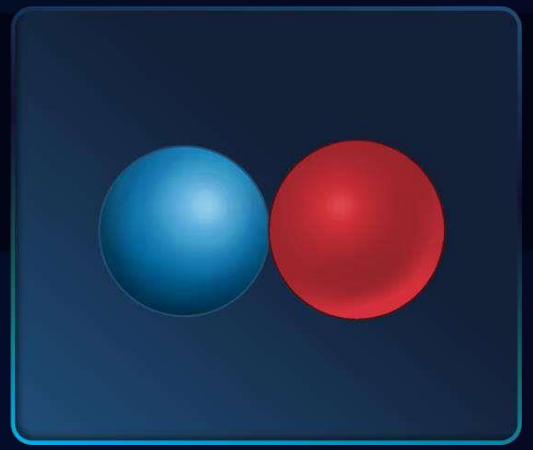


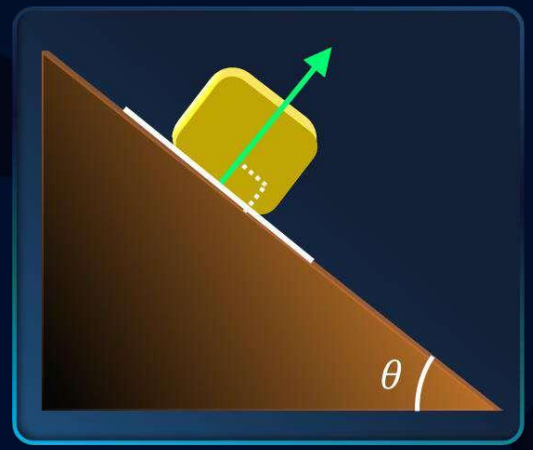
Parallel component

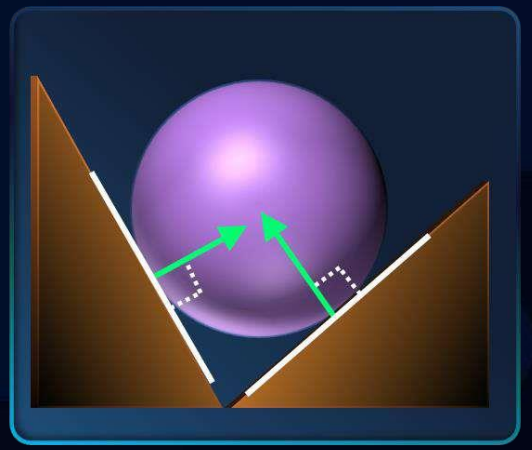
Draw the normal reaction for the following cases.

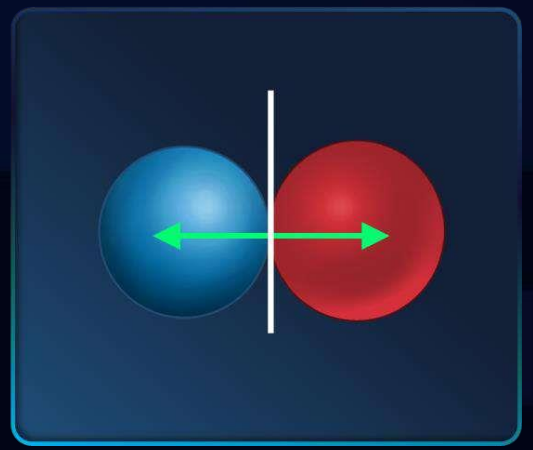




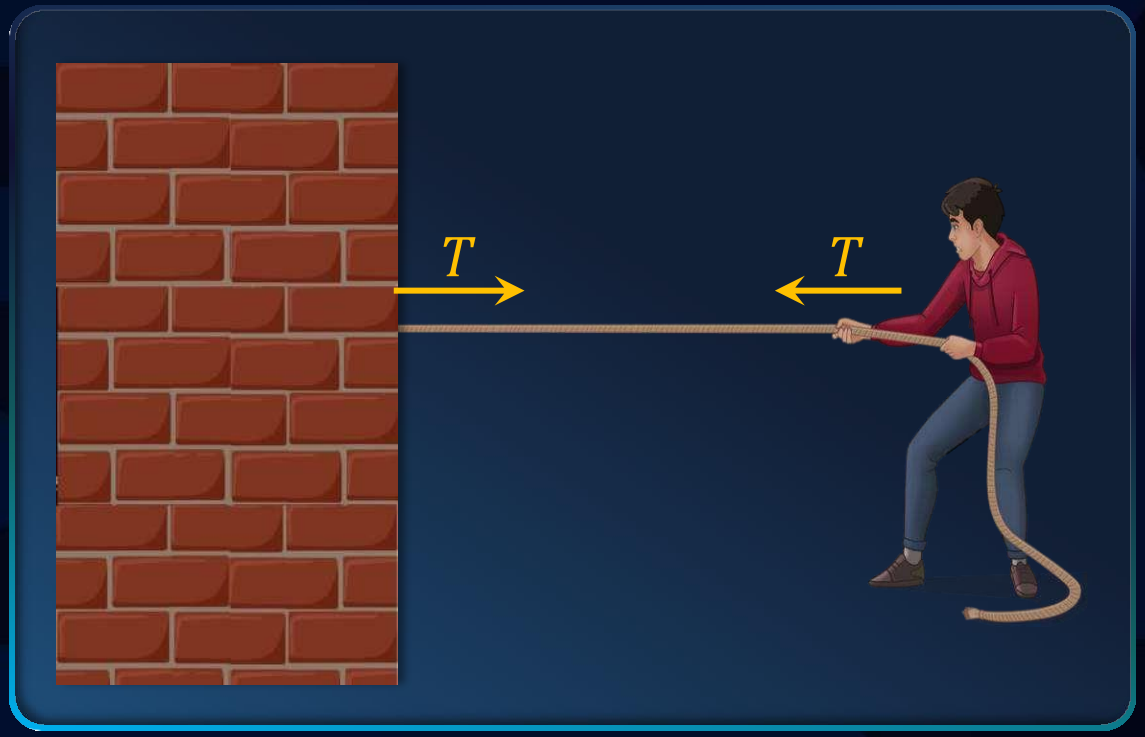








Tension



- The resistance of a string or a rope against any change in its original composition is called tensile force or tension.
- Tension always acts away from the tied ends.
- Tension is always pulling in nature.

Forces

Contact Forces

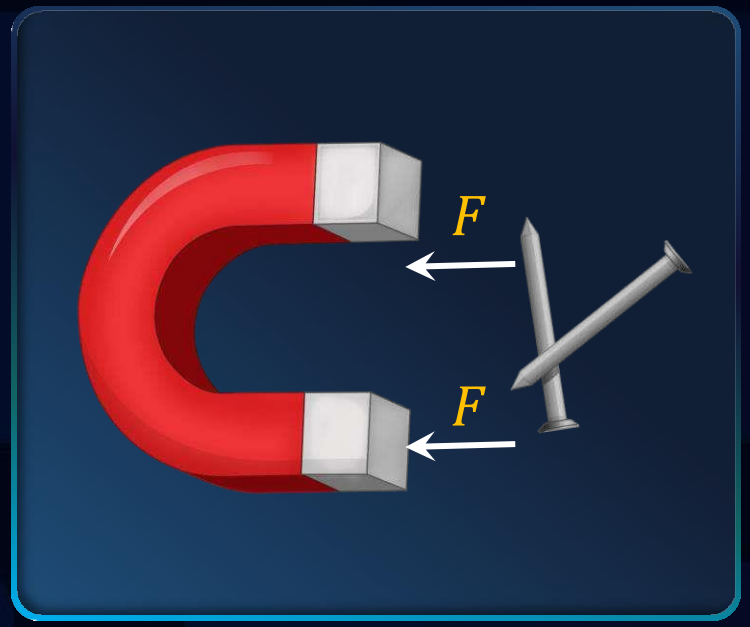
Non-Contact Forces



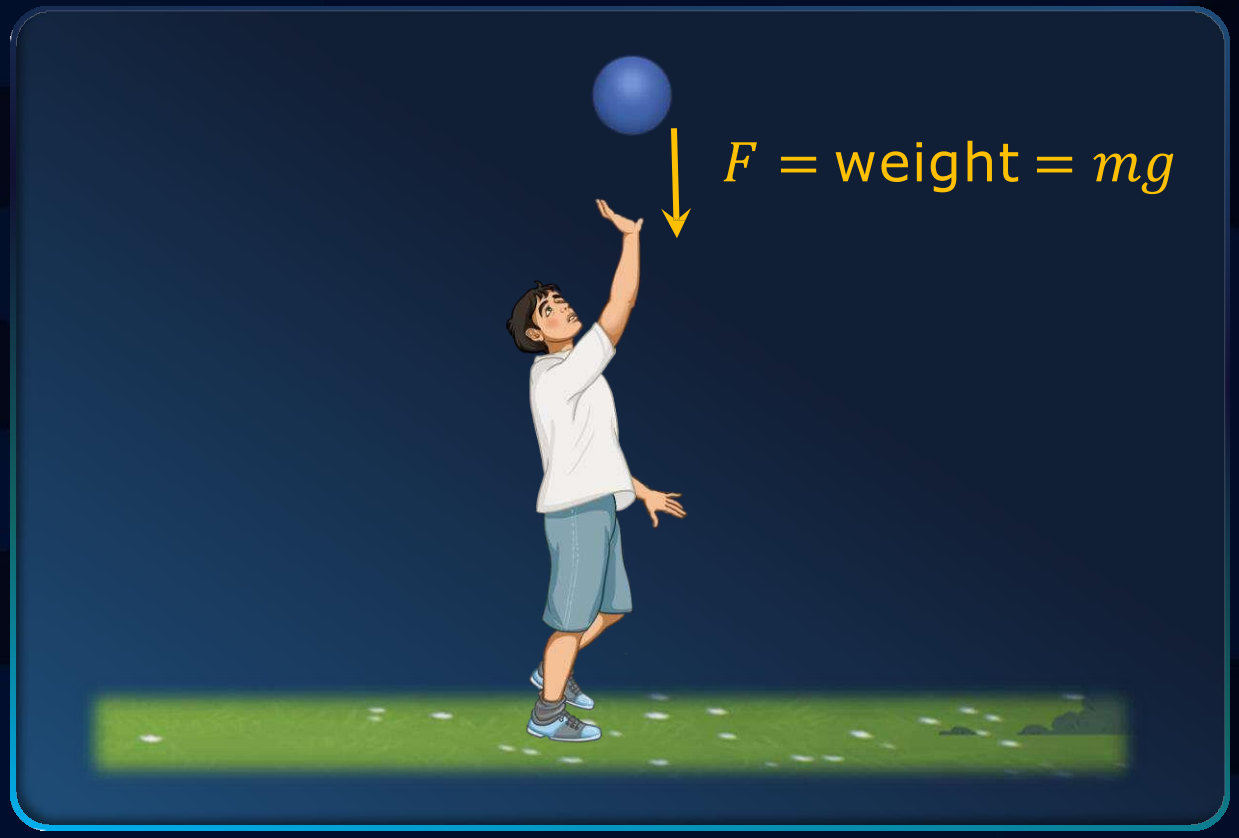
Non - Contact Forces

- A Non-Contact force is a force which acts on an object without coming physically in contact with it, e.g., gravitational force





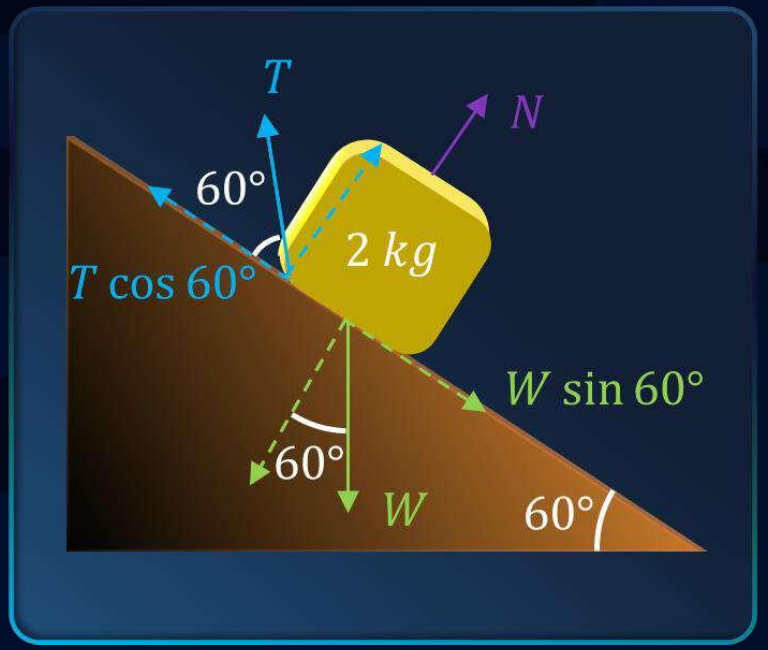
Weight



- The weight of an object of mass m near the surface of the Earth is mg.($g = 9.8\ m/s^2$)
- It is always directed towards centre of the earth.

A block of mass $2\ kg$ on a frictionless plane inclined at an angle $60°$ with the horizontal is supported using a string attached to the body and held at and angle $60°$ to the plane. Compute the tension in the string if the block is stationary.($g = 10\ m/s^2$)

Solution :



As the block is stationary, $F_{net} = 0$

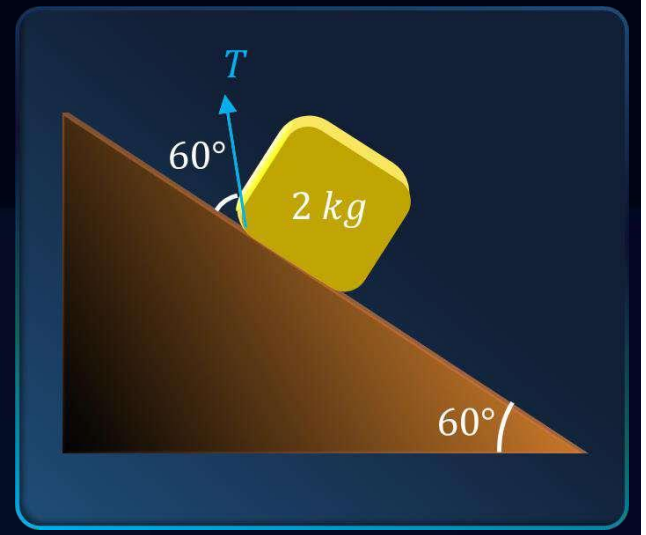
$F = 0$

$W \sin 60° = T \cos 60°$

$$T = \frac{W \sin 60°}{\cos 60°}$$

$$T = \frac{mg\sqrt{3}}{1}$$

$$T = 20\sqrt{3}\ N$$

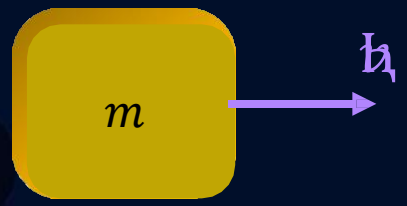


Momentum

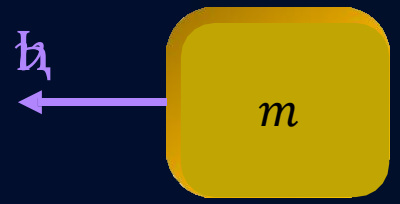
Momentum is a quantity possessed by an object by virtue of its mass and motion.
It is also known as the quantity of motion.

- It is a vector quantity.
- SI Unit : $kg\ m\ s^{-1}$

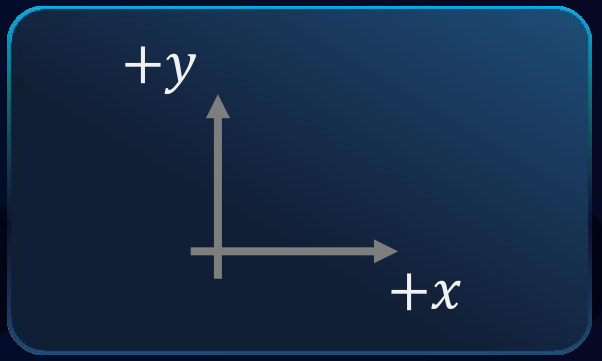
$$\vec{p} = m\vec{v}$$



$$\vec{p} = +mv\ \hat{\imath}$$



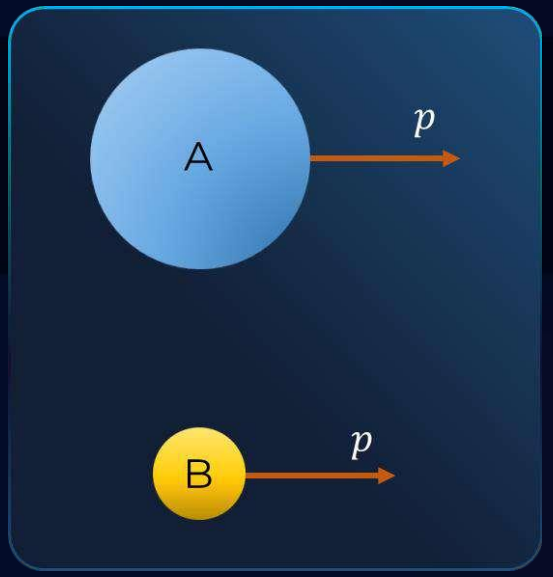
$$\vec{p} = -mv\ \hat{\imath}$$



Two bodies having masses $6\ kg$ and $9\ kg$ are said to have equal momentum while in motion. Find out the ratio of their velocities.

Solution : Given : $m_1 = 6\ kg, m_2 = 9\ kg$

To find : $\frac{v_1}{v_2} = ?$



Solution : $\because\ p = mv = constant$

$$\Rightarrow \frac{p_1}{p_2} = \frac{m_1 v_1}{m_2 v_2} \qquad [\because p_1 = p_2 = p\ ]$$

$$\Rightarrow 1 = \frac{6 \times v_1}{9 \times v_2}$$

$$\therefore \frac{v_1}{v_2} = \frac{9}{6} = \frac{3}{2}$$

a. $\frac{2}{3}$

b. $\frac{1}{3}$

c. $\frac{1}{2}$

d. $\frac{3}{2}$

Newton's Second Law of Motion

- The rate of change of momentum of an object is proportional to the net external force and is in the direction of net force applied.

$$\vec{F} \propto \frac{d\vec{p}}{dt}$$

$$\vec{F} = k\frac{d\vec{p}}{dt}$$

$$= k\frac{d(m\vec{v})}{dt}$$

$$= km\frac{d\vec{v}}{dt} \quad [\text{Assuming mass, } m = \text{ constant and } k = 1]$$

$$\vec{F} = m\vec{a}$$

$$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$$

Along x – axis

$$F_x = \frac{dp_x}{dt} = ma_x$$

Along y – axis

$$F_y = \frac{dp_y}{dt} = ma_y$$

Along z – axis

$$F_z = \frac{dp_z}{dt} = ma_z$$

Newton's second law of motion is applicable only for point masses.

Momentum of a particle is given by $\vec{p} = 5t^2\hat{i} + 10t\hat{j}$. Find the magnitude and direction of force acting on the particle at $t = 1\ s$.

Given : $\vec{p} = (5t^2\hat{i} + 10t\hat{j})$, $t = 1\ s$

To find : $|\vec{F}| = ?, \theta = ?$

Solution : $\because \vec{F} = \frac{d\vec{p}}{dt}$

$\Rightarrow \vec{F} = \frac{d}{dt}(5t^2\hat{i} + 10t\hat{j})$

$\Rightarrow \vec{F} = 10t\hat{i} + 10\hat{j}$

$\Rightarrow \vec{F} = 10(1)\hat{i} + 10\hat{j}$

$\Rightarrow \vec{F} = 10\hat{i} + 10\hat{j}$

$\Rightarrow \vec{F} = \sqrt{(10)^2 + (10)^2}$

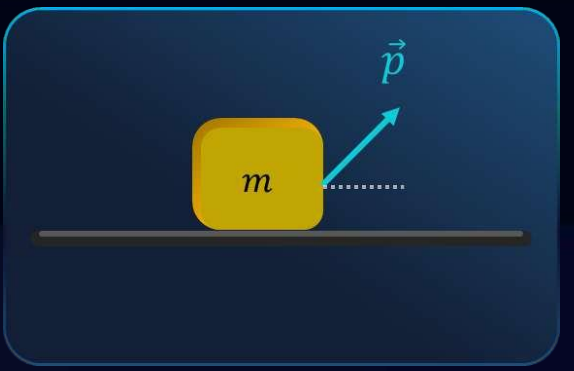
$\Rightarrow |\vec{F}| = 10\sqrt{2}\ N$

$\because \tan\theta = \frac{F_y}{F_x}$

$\Rightarrow \tan\theta = \frac{10}{10} = 1$

$\Rightarrow \theta = \tan^{-1}(1)$

$\Rightarrow \theta = 45^\circ$



a. $10\sqrt{2}\ N, 45^\circ$

b. $10\sqrt{2}\ N, 30^\circ$

c. $20\sqrt{2}\ N, 45^\circ$

d. $20\sqrt{2}\ N, 30^\circ$

Unit of Force

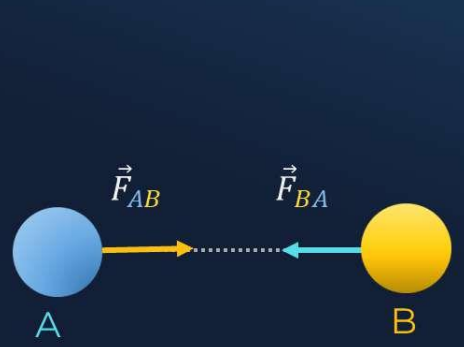
S.I. Unit of Force : N(newton)

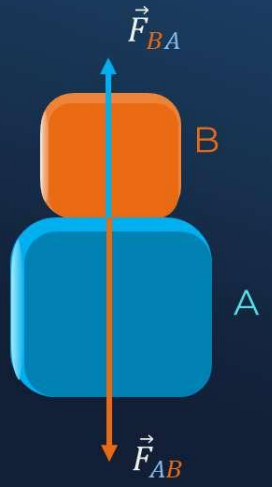
Gravitational Unit of Force : $kg - weight$



$$W = mg$$

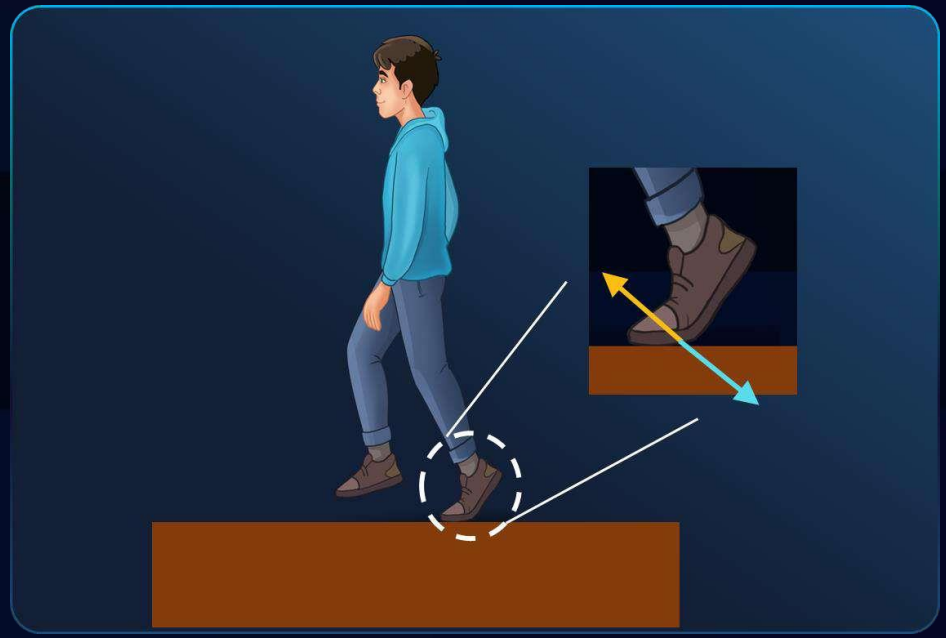
Newton’s Third Law of Motion





$\vec{F}_{BA}$ = Force on B by A

$\vec{F}_{AB}$ = Force on A by B

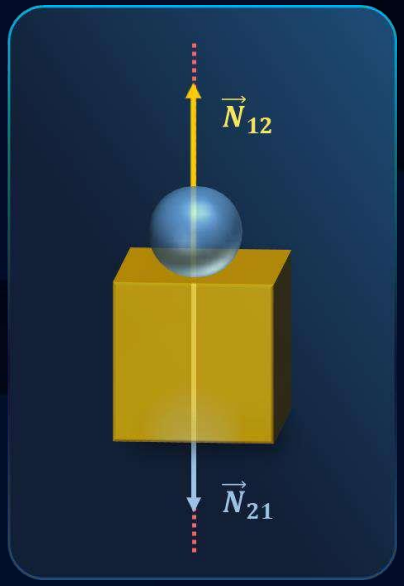


- To every action, there is always an equal and contrary reaction (or) the mutual actions of any two bodies are always equally and oppositely directed.
- Forces always occur in pairs i.e., action-reaction pair. Both act at the same instant.

Action-Reaction Pair

- Forces always occur in pairs i.e., action-reaction pair. Both act at the same instant.





Conditions for Action – Reaction pair

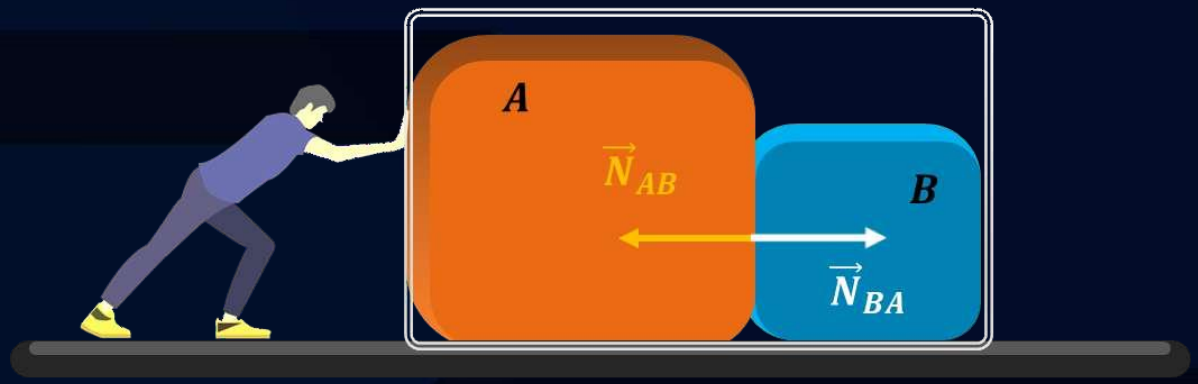
Action – Reaction Pairs

- Occur simultaneously
- Act in Opposite directions
- Act along the same line
- Act on different objects
- Equal magnitude and same nature

System

- Two and more objects which interact with each other form a system.
- Action-reaction pairs existing within the system are treated as internal forces.
- Forces exerted on the constituents of a system by the surroundings are called external forces.

SYSTEM-1



$\vec{N}_{AB}$ & $\vec{N}_{BA}$ are internal forces

- Note : All surfaces considered are smooth.

A particle moving with velocity v is acted by three forces shown by the vector triangle PQR. The velocity of the particle will _____

Given : Force acting are $\overrightarrow{PQ}, \overrightarrow{QR}$ and $\overrightarrow{PR}$

To find : $v = ?$

Solution : For, a closed polygon,

$\because \vec{F}_{net} = ma$

$\Rightarrow \sum \vec{F} = 0$ (Cyclic order) $\left[\because \sum \vec{F} = 0\right]$

$\Rightarrow 0 = m\frac{dv}{dt}$

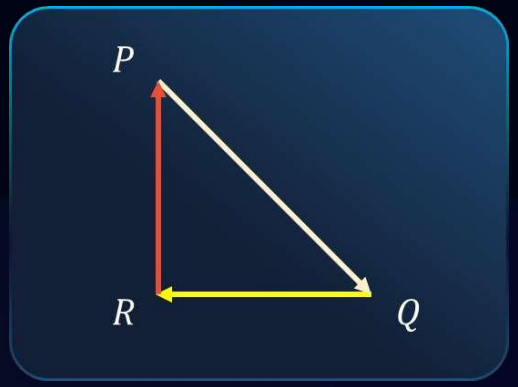
$\Rightarrow \vec{F}_{PR} + \vec{F}_{PQ} + \vec{F}_{QR} = 0$

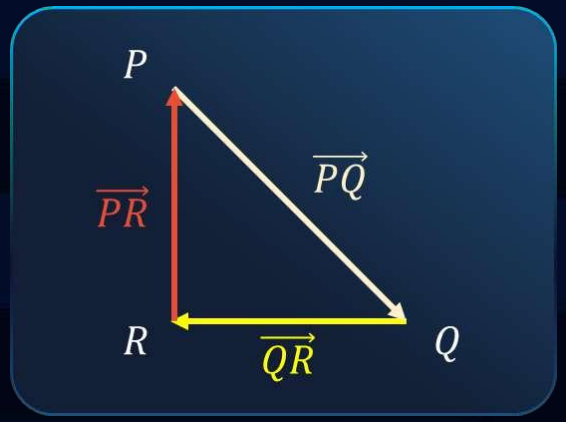
$\Rightarrow \frac{dv}{dt} = 0$

$\Rightarrow \vec{F}_{PR} + \vec{F}_{PQ} = -\vec{F}_{QR}$

$\Rightarrow v = constant$

$\Rightarrow \vec{F}_{PR} + \vec{F}_{PQ} = \vec{F}_{RQ}$





a. Decrease

b. Remain constant

c. Changes according to the smallest force QR

d. Increase

Conservation of Linear Momentum

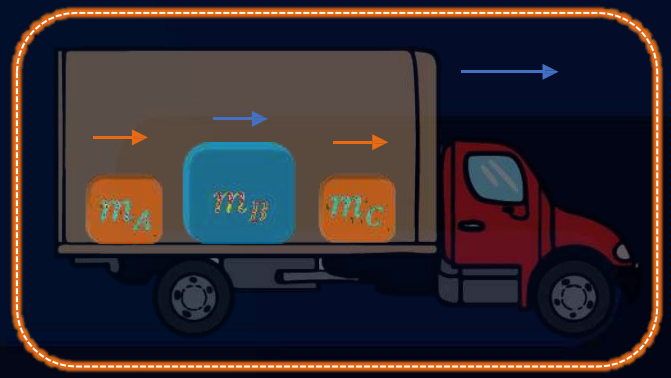
If the external forces acting on the system add up to zero, the linear momentum of a system remains constant. This is known as the principle of conservation of linear momentum (PCLM).

$$\Rightarrow \vec{F}_{net} = \left(\sum \vec{F}_{ext}\right)_{sys} = \frac{d\vec{p}_{sys}}{dt}$$

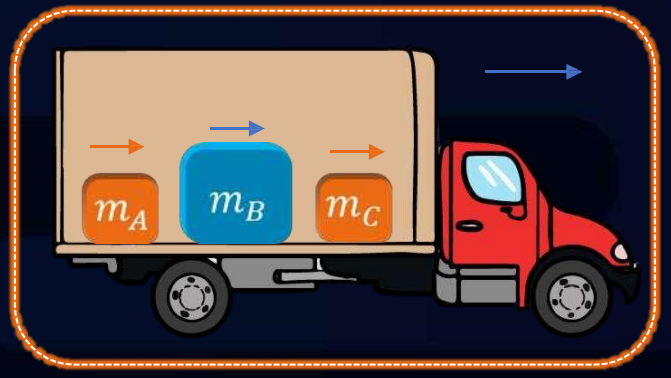
$$\Rightarrow \left(\sum \vec{F}_{ext}\right)_{sys} = 0$$

$$\vec{p}_{initial} = \vec{p}_{final}$$

$\vec{p}_{initial} = M\vec{v}_{COM}$



$\vec{p}_{final} = M\vec{v}_{COM}$



Conservation of Linear Momentum

$$\left(\sum \vec{F}_{ext}\right)_{system} = 0 \Rightarrow (\vec{p}_i)_{system} = (\vec{p}_f)_{system}$$

If $\left(\sum \vec{F}_{ext}\right)_x = 0$

$\Rightarrow (\vec{p}_i)_x = (\vec{p}_f)_x$

Momentum is conserved along $x-axis$

If $\left(\sum \vec{F}_{ext}\right)_y = 0$

$\Rightarrow (\vec{p}_i)_y = (\vec{p}_f)_y$

Momentum is conserved along $y-axis$

If $\left(\sum \vec{F}_{ext}\right)_z = 0$

$\Rightarrow (\vec{p}_i)_z = (\vec{p}_f)_z$

Momentum is conserved along $z-axis$

A body of mass $1000\ kg$ is moving horizontally with a velocity $50\ m\mathrm{T}s$. A mass of $250\ kg$ is added suddenly. Find the final velocity.

Solution :

Given: Mass of body $= 1000\ kg$, Mass added $= 250\ kg$, $v_i = 50\ m/s$

To find: $v_f = ?$

Solution: $\Rightarrow m_1 v_1 = m_2 v_2$

$\Rightarrow 1000 \times 50 = (1000 + 250) v_2$

$\Rightarrow v_2 = \dfrac{50000}{1250}$

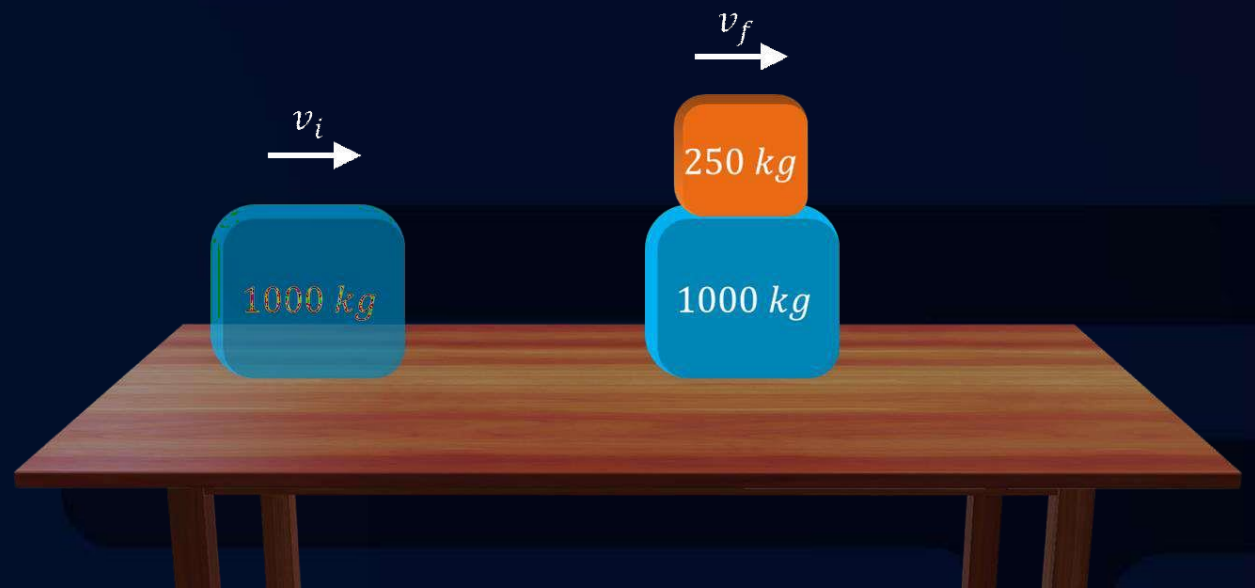
$v_2 = 40\ m\mathrm{T}s$

a. $40\ m/s$

c. $20\ m/s$

b. $30\sqrt{2}\ m/s$

d. $50\ m/s$



A gun of mass $10\ kg$ fires 4 bullets per second. The mass of each bullet is $20\ g$ and the velocity of the bullet when it leaves the gun is $300\ m/s$. The average force per second required to hold the gun while firing is-

Solution :

Given: $m_B = 20\ g, v_B = 300\ m/s, M_G = 10\ kg$



To find: $F = ?$

Solution:

Rate of change of momentum of the bullet in forward direction = Force required to hold the gun

$\Rightarrow n(mv) = F$

$\Rightarrow F = 4 \times 20 \times 10^{-3} \times 300$

$\Rightarrow F = 24\ N$

a. $6\ N$

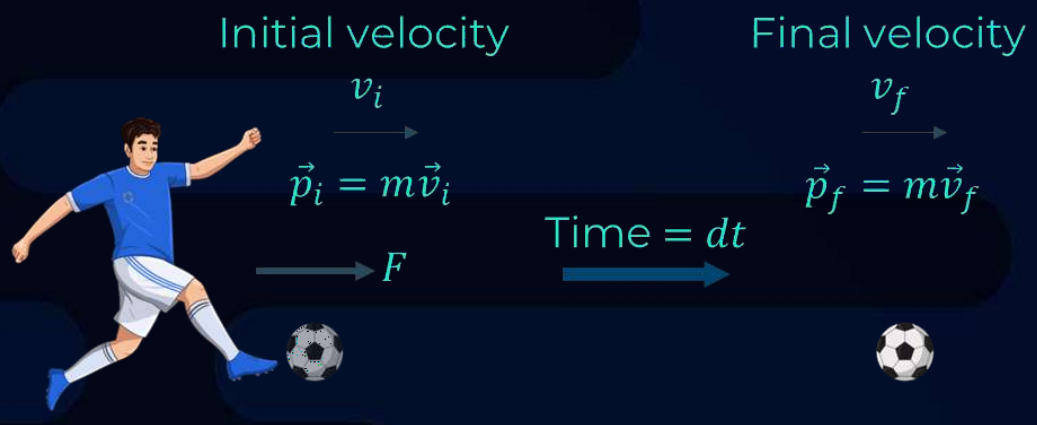
c. $24\ N$

b. $8\ N$

d. $240\ N$

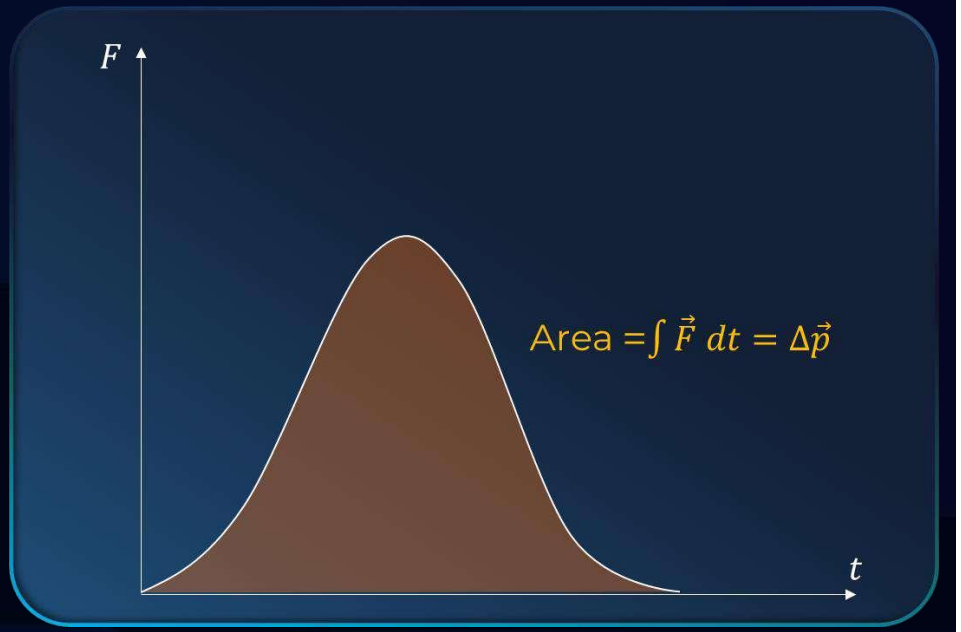
Impulse

The change in momentum suffered by a system due to action of a force ($\vec{F}$) for an infinitesimally small amount of time is the impulse ($\vec{J}$) given by the force ($\vec{F}$) to the system.



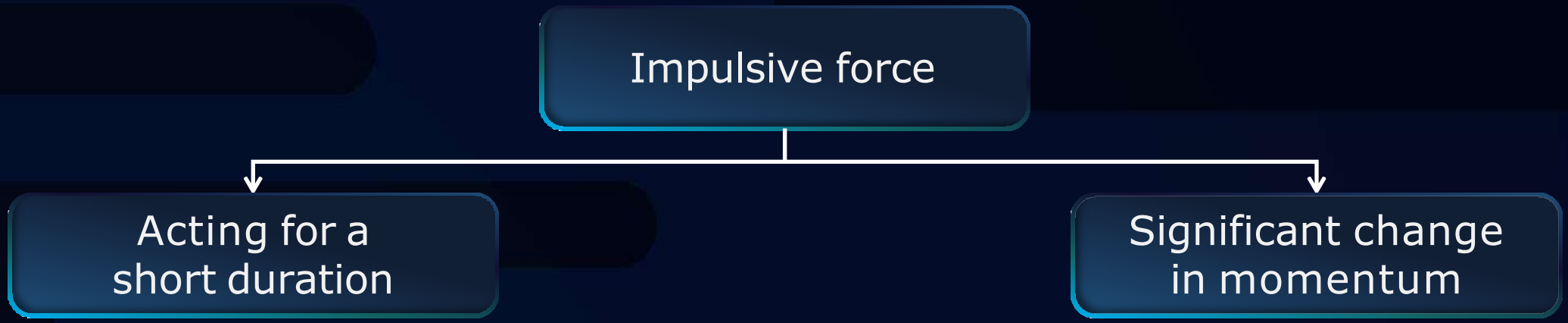
$$\vec{J} = \int \vec{F}\,dt = \vec{p}_f - \vec{p}_i = \Delta\vec{p}$$

Impulse applied to an object in a given time interval is the area under force time ($F - t$) graph for the same time interval.



Impulse

Force acting for a short interval of time is called an impulsive force.



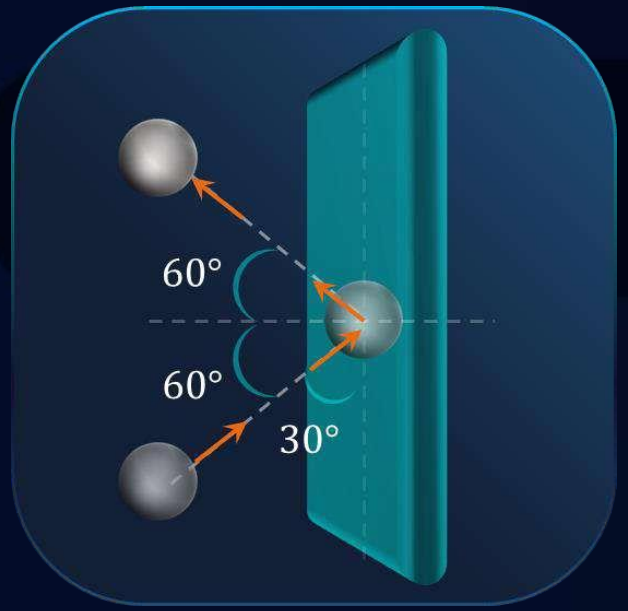
A $100\ g$ iron ball having a velocity $10\ m/s$ collides with a wall at an angle $30°$ and rebounds at the same angle. If the period of contact between the ball and wall is 0.1 seconds, then the force experienced by the wall is-

Solution :

Given: $m = 100\ g, v = 10\ m/s, \theta = 30°, t = 0.1\ s$

To find: $F = ?$

Solution: $\Rightarrow F = \dfrac{Change\ in\ momentum}{Time}$



$\Rightarrow F = \dfrac{2mv\ cos60°}{0.1}$

$\Rightarrow F = \dfrac{2 \times 0.1 \times 10 \times \frac{1}{2}}{0.1}$

$F = 10\ N$

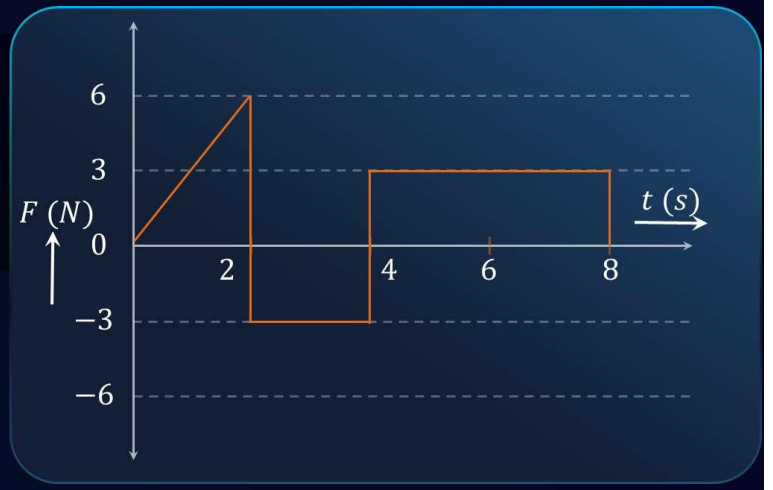
a. $10\ N$

c. $1\ N$

b. $100\ N$

d. $0.1\ N$

The force F acting on a particle of mass m is indicated by the force-time graph shown below. The change in momentum of the particle over the time interval from zero to $8\ s$ is-



Given: $\Delta t = 8\ s$

To find: $J = ?$

Solution:

Change in momentum (Δp) = Impulse

$\Rightarrow (\Delta p)$ = area under $F - t$ graph

$\Rightarrow (\Delta p) = \frac{1}{2} \times 2 \times 6 - 2 \times 3 + 4 \times 3$

$(\Delta p) = 12\ Ns$



a. $12\ Ns$

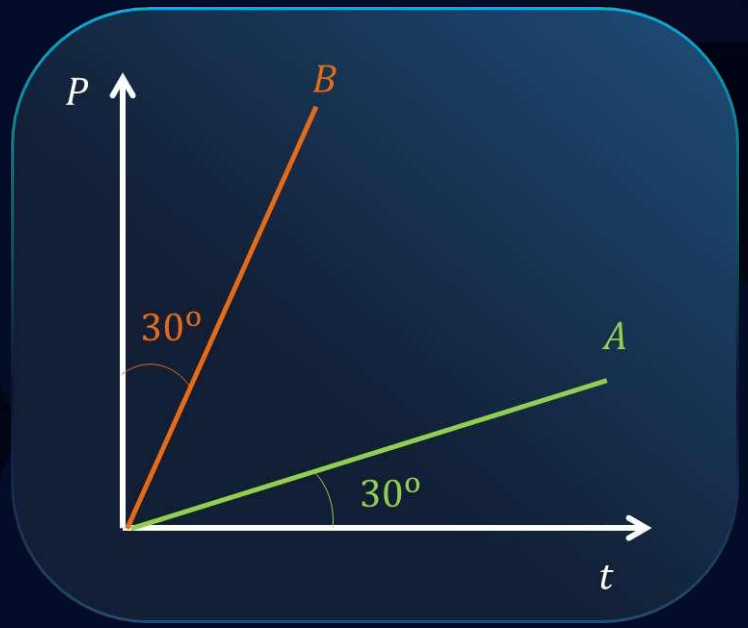
b. $6\ Ns$

c. $24\ Ns$

d. $20\ Ns$

The momentum-time graph of a particle is given below. Calculate $\frac{F_A}{F_B}$.

To find: $\frac{F_A}{F_B} = ?$



Solution: $\Rightarrow \frac{F_A}{F_B} = \frac{\tan 30^o}{\tan 60^o} = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}}$

$$\frac{F_A}{F_B} = \frac{1}{3}$$

a. 1:2

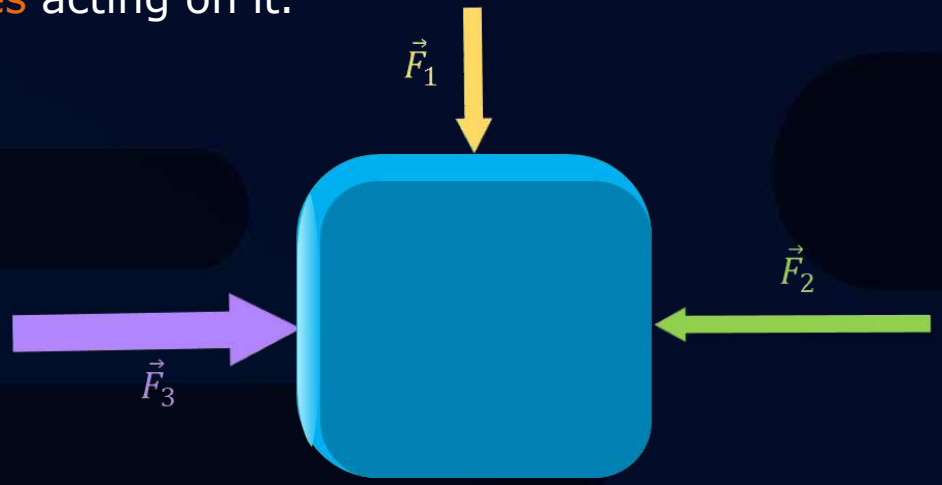
b. 1:3

c. 3:1

d. 2:1

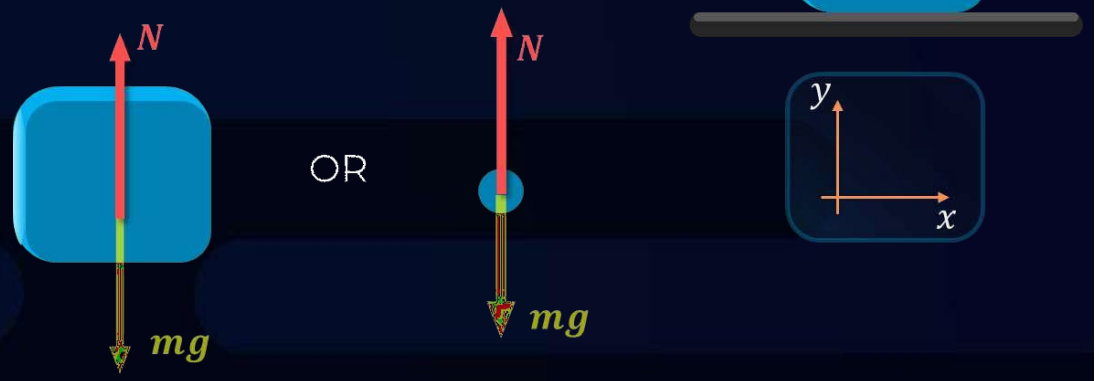
Free Body Diagram

- Diagrammatic representation of a body which is isolated from its surroundings showing all the external forces acting on it.



Steps to draw FBD

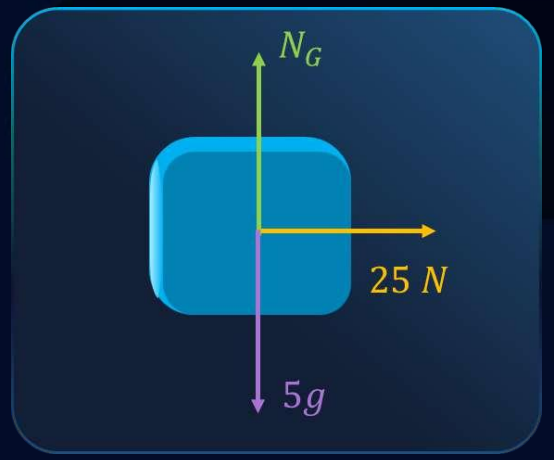
- Identify the system.
- Draw all the external forces acting on the system. (block B of mass m)
- Choose axes and resolve the forces along them.

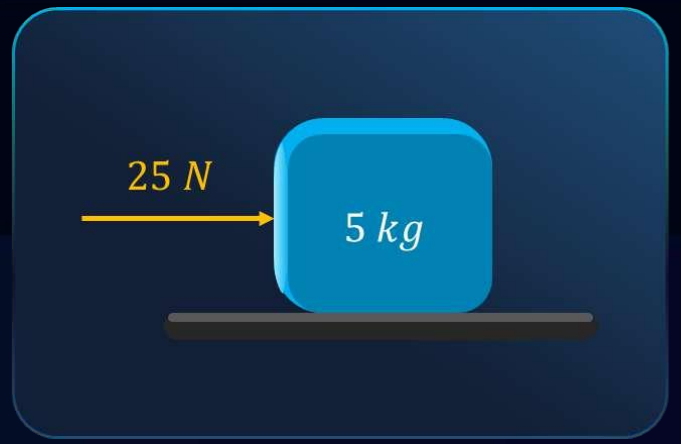


A block of mass $5\ kg$ is kept on a smooth surface. A force of $25\ N$ is applied as shown. Find the acceleration of the block.

Solution : Along y-axis :

$\Sigma F_y = N - mg = 0$





$a_y = 0$

Along x-axis :

$\Sigma F_x = 25\ N$

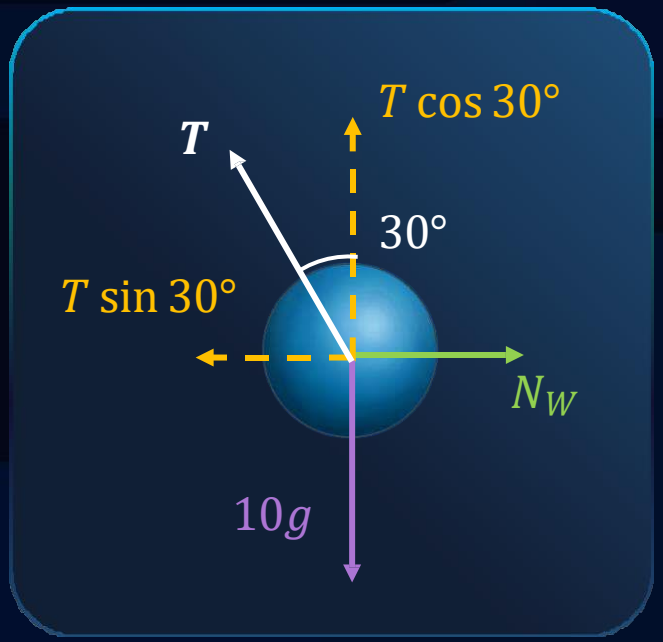
$ma_x = 25\ N$

$a_x = \frac{25}{5}$

$a_x = 5\ m/s^2$

Equilibrium

- When the net force on the system is zero, the object is said to be in the state of equilibrium.



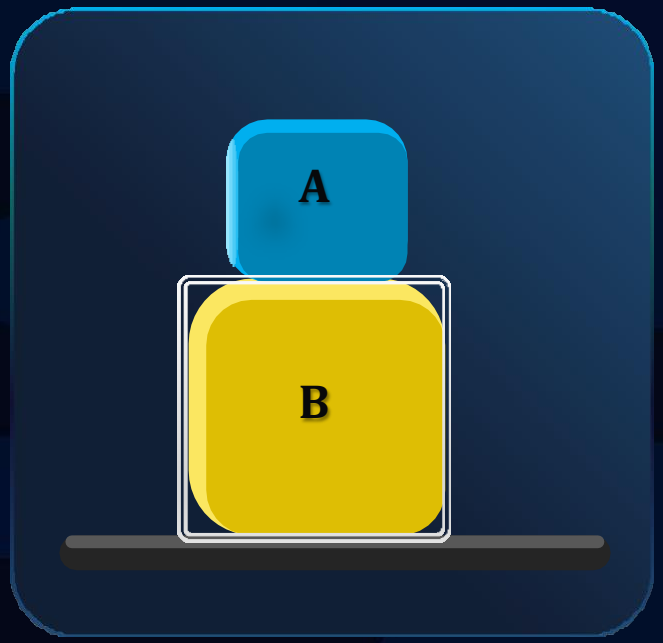
$$\left(\sum F_{ext}\right)_{sys} = 0$$

$$\sum F_x = 0$$

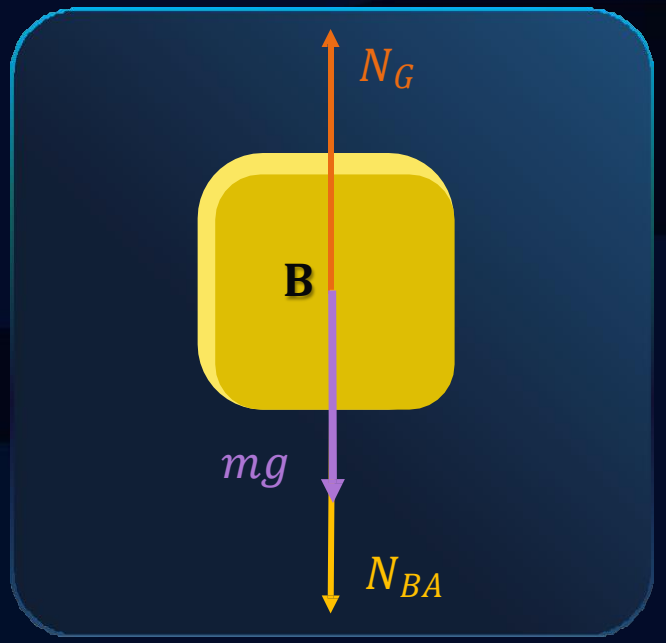
$$\sum F_y = 0$$

$$\sum F_z = 0$$

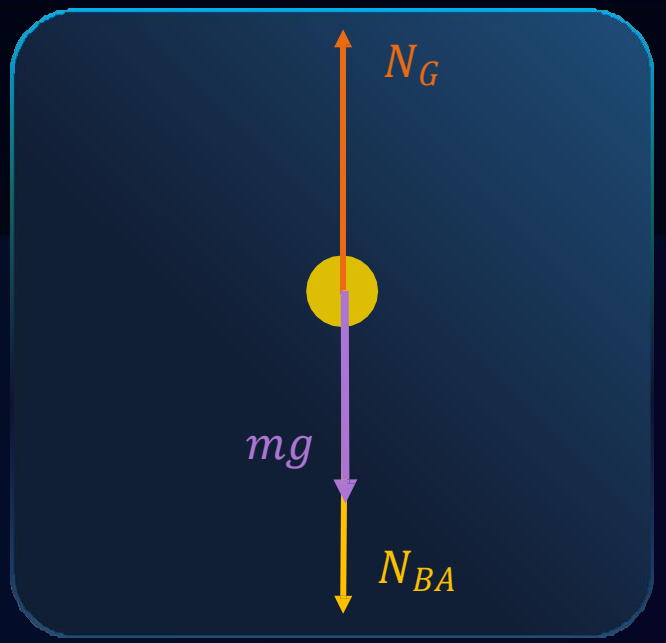
FBD :Two blocks in contact



Isolate the system



Draw the external forces



Choose the axes and resolve

10 coins each of mass m are placed one over the other. Coins are numbered from the bottom. Calculate-

i) Force exerted on the 4th coin on all the coins above it.

ii) Force exerted by 6th coin on all the coins below it.

Solution :

Given : Mass of each coin = m





i) Force on 4th coin by all the coins above it

Force on 4th coin = Force by the 4th coin

Therefore, Force on the 4th coin = $6mg$



ii) Force by 6th coin on all the coins below it

Force by 6th coin = $4mg + mg$

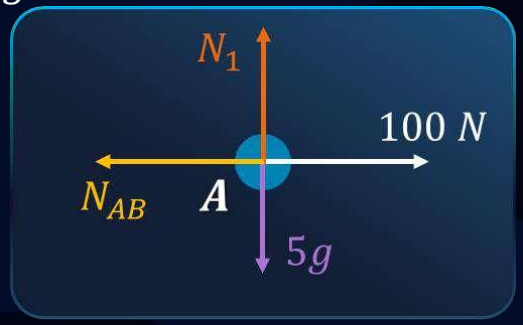
Therefore, Force by the 6th coin = $5mg$

Two blocks kept in contact experience $100\ N$ force as shown. Find the acceleration of the $5\ kg$ block and the normal force between the blocks.

Given : $F = 100\ N, m_A = 5\ kg, m_B = 20\ kg$

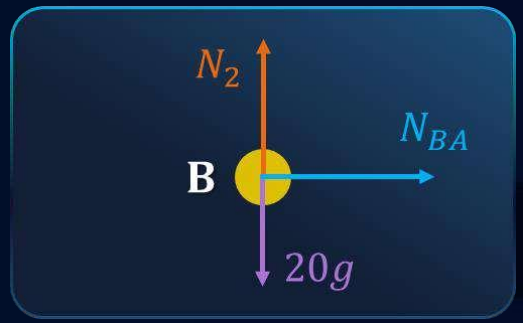
To find : a_A and $N_{AB} = N_{BA}$

Solution :



i) FBD of block A

$F_x = 100 - N_{AB} = 5a_x$ ---------- 1



ii) FBD of block B

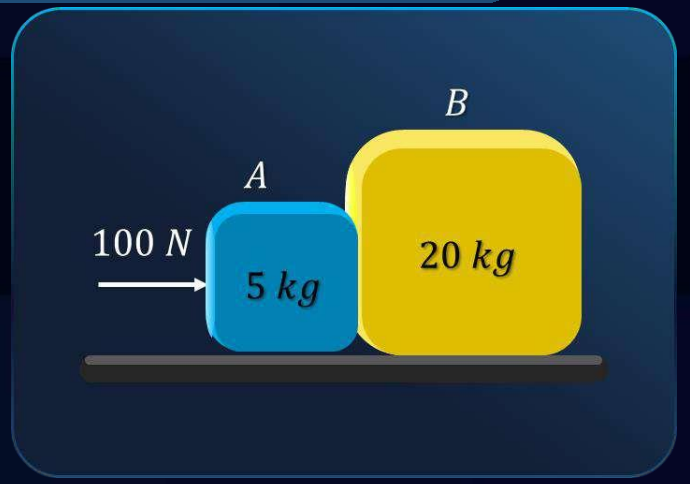
$F_x = N_{BA} = 20a_x$ ---------- 2

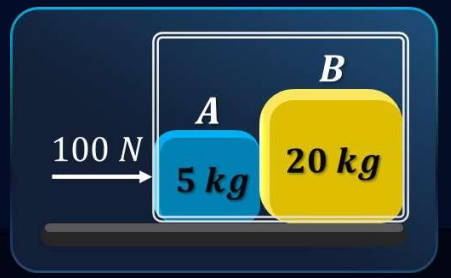
From 1and 2, $100 = 25a_x$

$a_x = 4\ m/s^2$

$N_{BA} = 80\ N$

Substituting in 2,

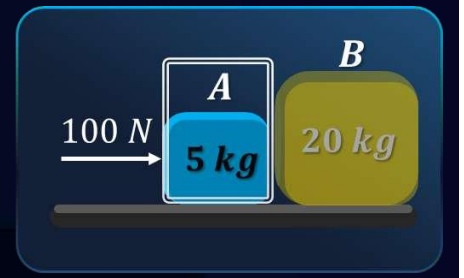




$F_{net} = ma_x$

$100 = 25a_x$

$a_x = 4\ m/s^2$



$\sum F_x = 100 - N_{AB} = 5a_x$

$100 - N_{AB} = 5 \times 4$

$N_{AB} = 80\ N$

Blocks Connected by a String

- Mass of string is negligible.
- String is inextensible/inelastic.

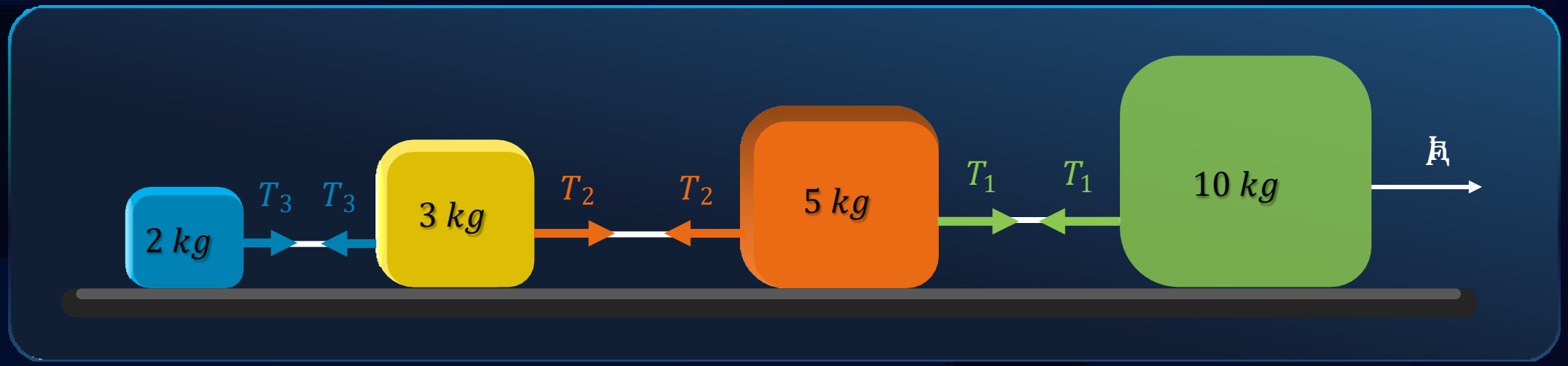
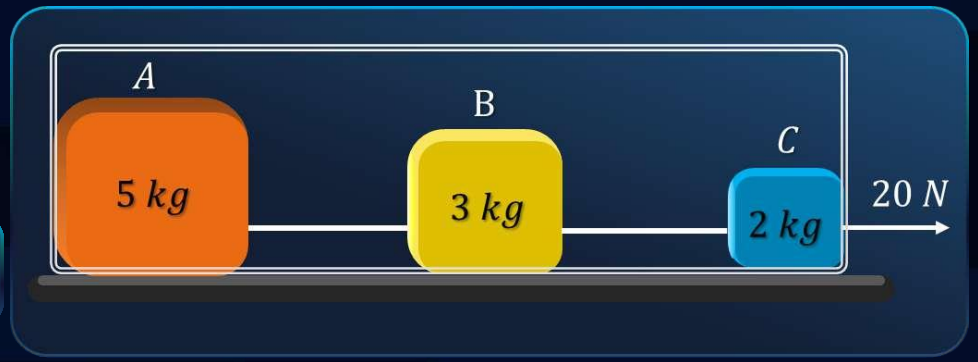


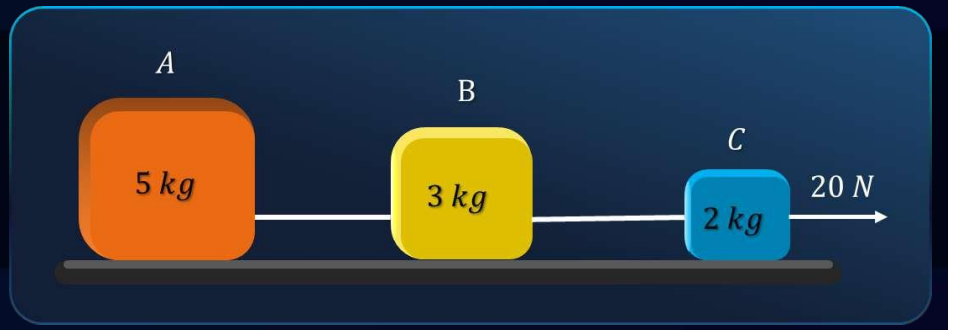
Figure shows three blocks connected by a light inextensible string. A pulling force of $20\ N$ is applied on the block of $2\ kg$ horizontally. Find the tension in the string connecting the three blocks.

Solution : $F_{net} = ma_x$

$20 = 10a_x$

$a_x = 2\ m/s^2$

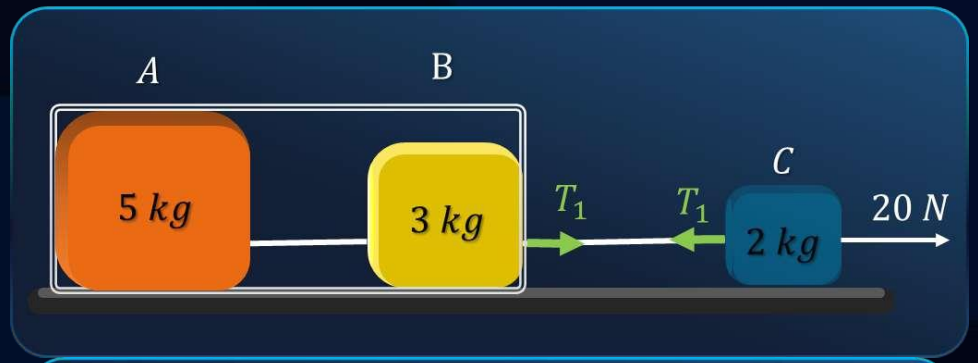




$F_x = T_1 = 8a_x$

$T_1 = 8 \times 2$

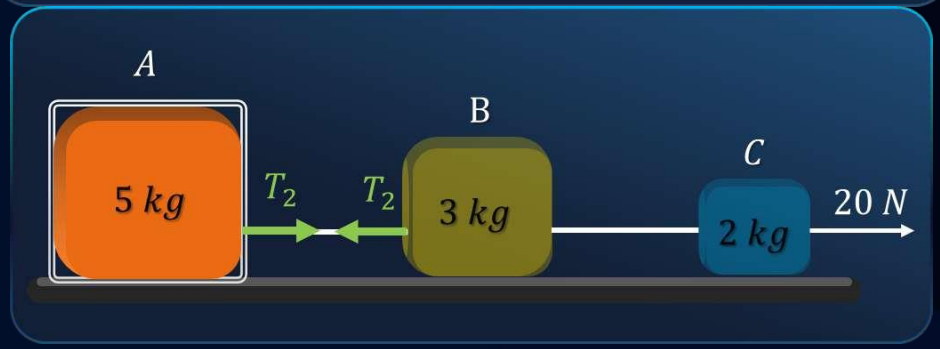
$T_1 = 16\ N$



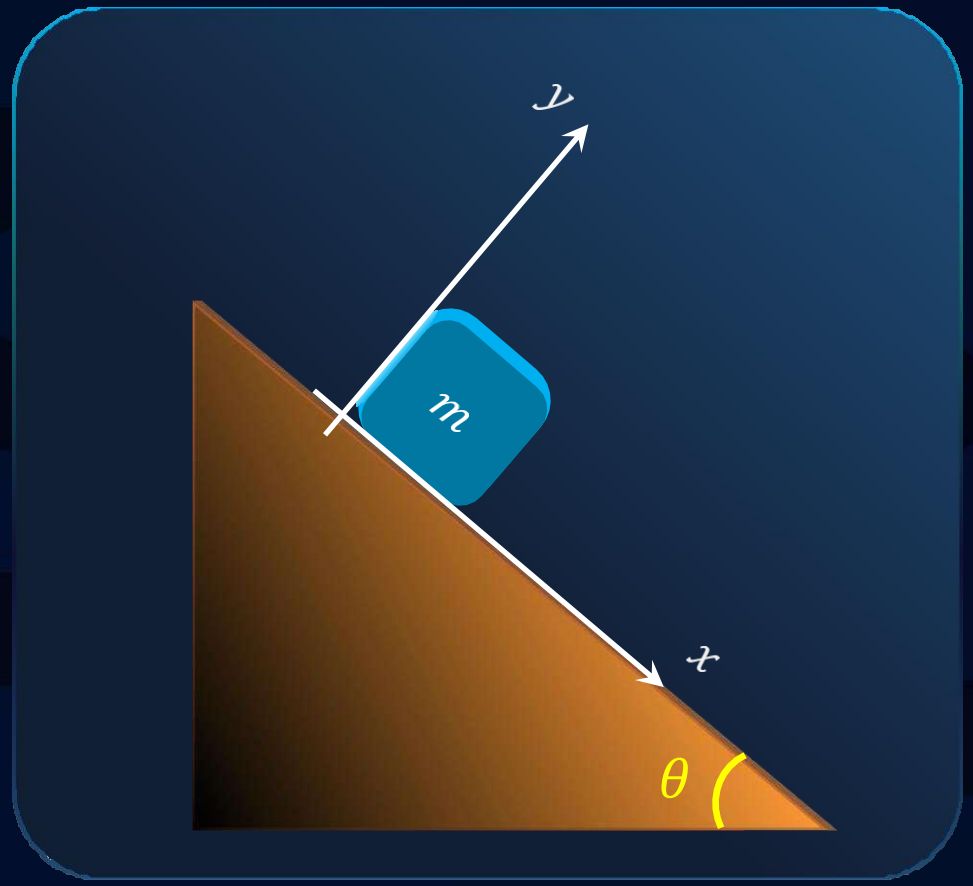
$F_x = T_2 = 5a_x$

$T_2 = 5 \times 2$

$T_2 = 10\ N$



Blocks on an Inclined Plane



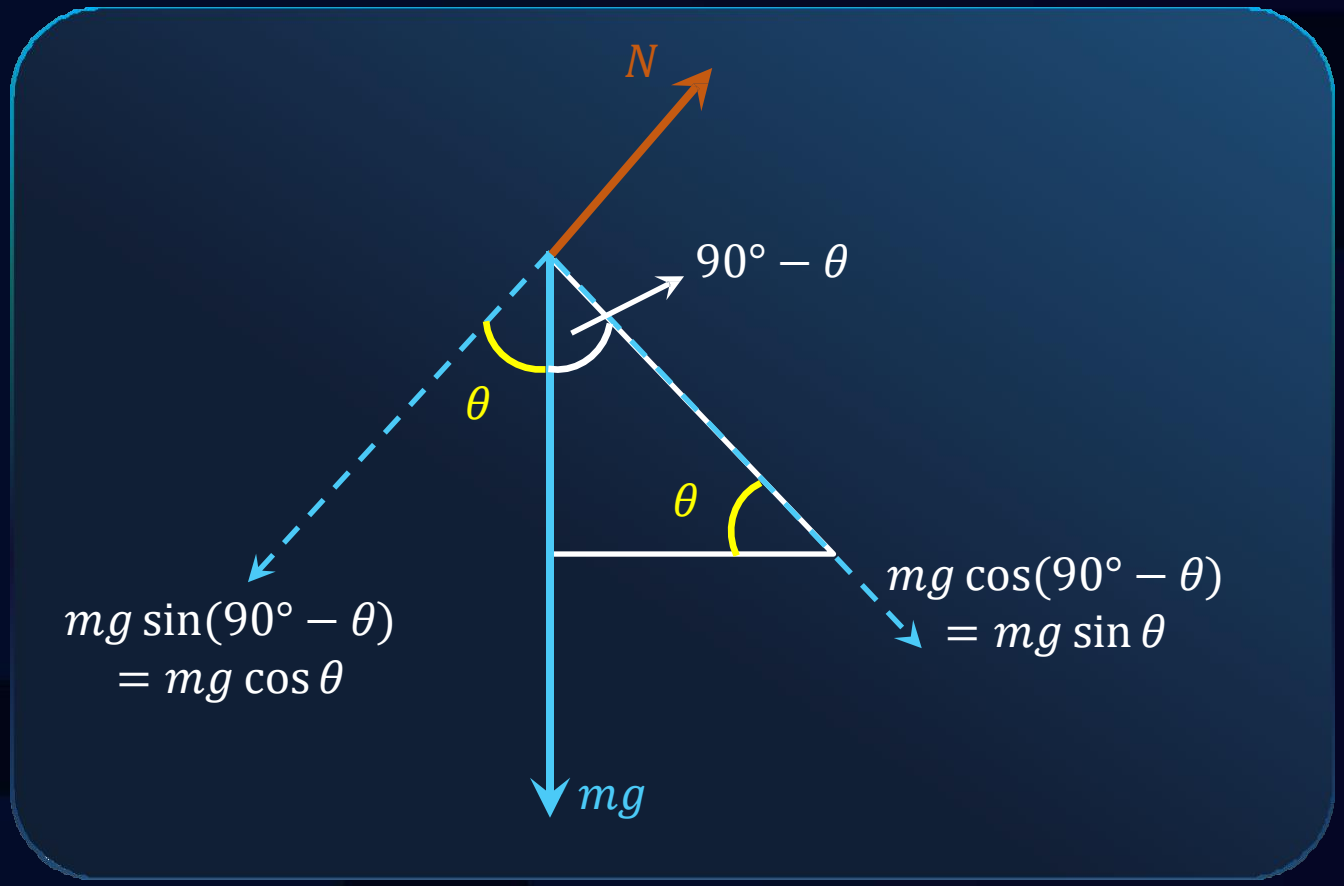


Figure shows three blocks connected by a light inextensible string. A pulling force of $100\ N$ is applied on the block of $2\ kg$ horizontally. Find the ratio of the tensions in the string connecting the three blocks. (Take $g = 10\ m/s^2$ and assume table to be smooth surface.)

Solution :

$$F_{net} = ma_x$$

$$100 - (m_A + m_B + m_C)g\sin 53° = 10a_x$$

$$100 - \left(10 \times 10 \times \frac{4}{5}\right) = 10a_x$$

$$20 = 10a_x$$

$$a_x = 2\ m/s^2$$

$$T_1 - (m_A + m_B)g\sin 53° = (m_A + m_B)a_x$$

$$T_1 - \left(8 \times 10 \times \frac{4}{5}\right) = 8 \times 2$$

$$T_1 = 16 + 64$$

$$T_1 = 80\ N$$

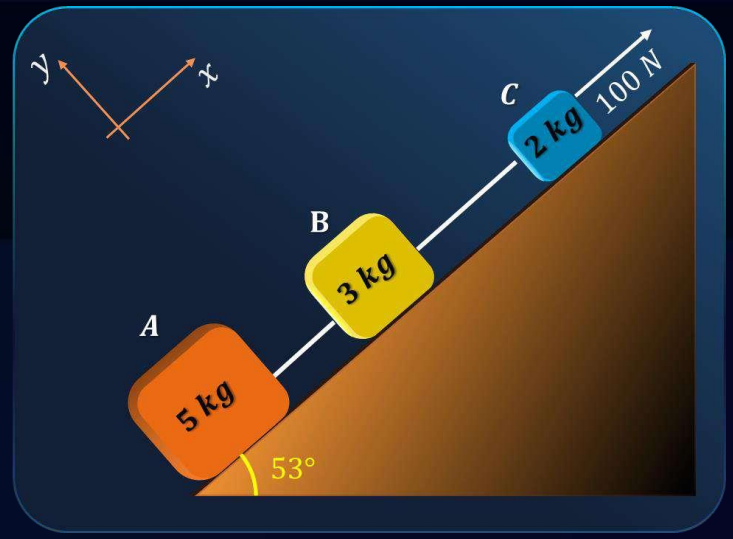
$$T_2 - m_A g\sin 53° = m_A a_x$$

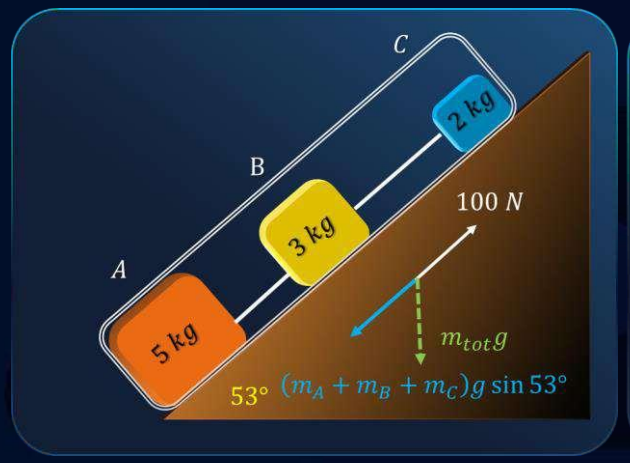
$$T_2 - \left(5 \times 10 \times \frac{4}{5}\right) = 5 \times 2$$

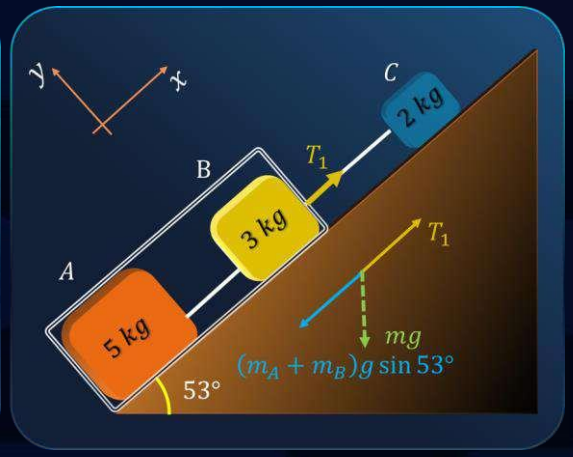
$$T_2 = 10 + 40$$

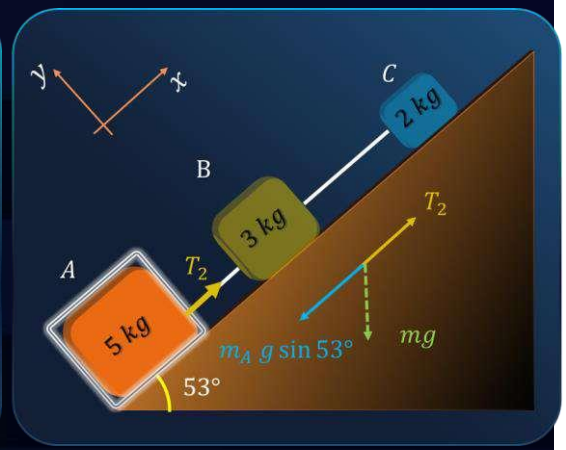
$$T_2 = 50\ N$$

$$T_1 : T_2 = 8 : 5$$



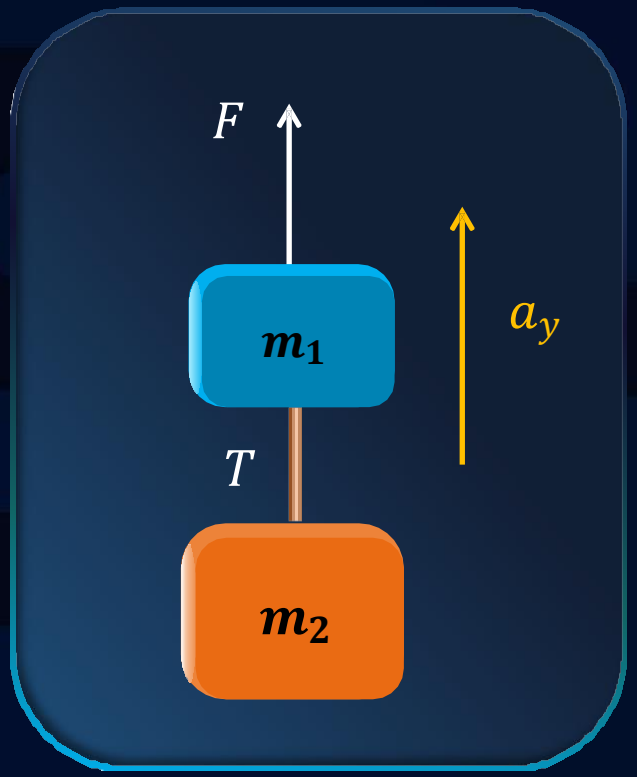


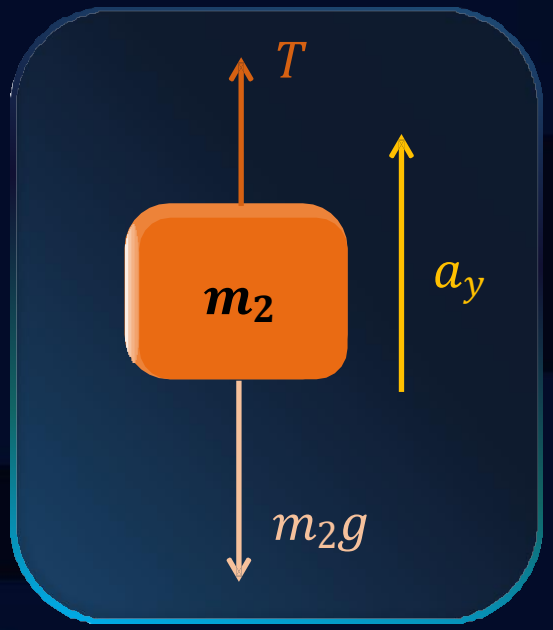




Vertical motion of the block

In Upward Direction:

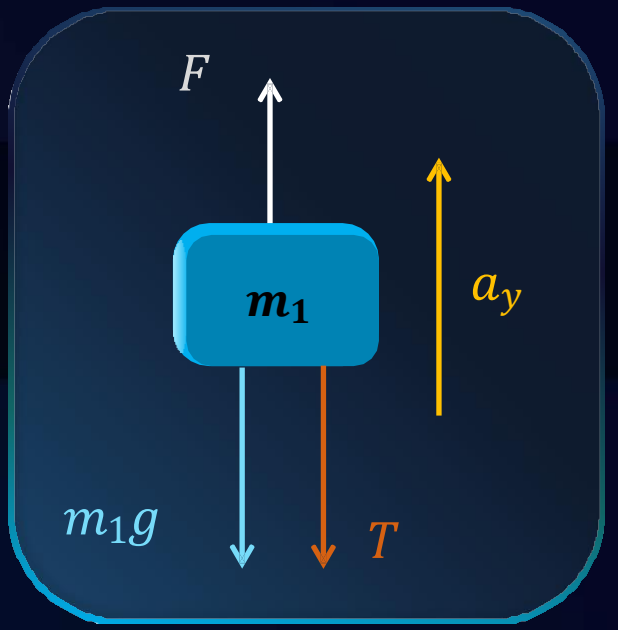




$$\Rightarrow T - m_2g = m_2a_y$$

$$\Rightarrow T = m_2(g + a_y)$$

$$T = m_2g_{eff}$$



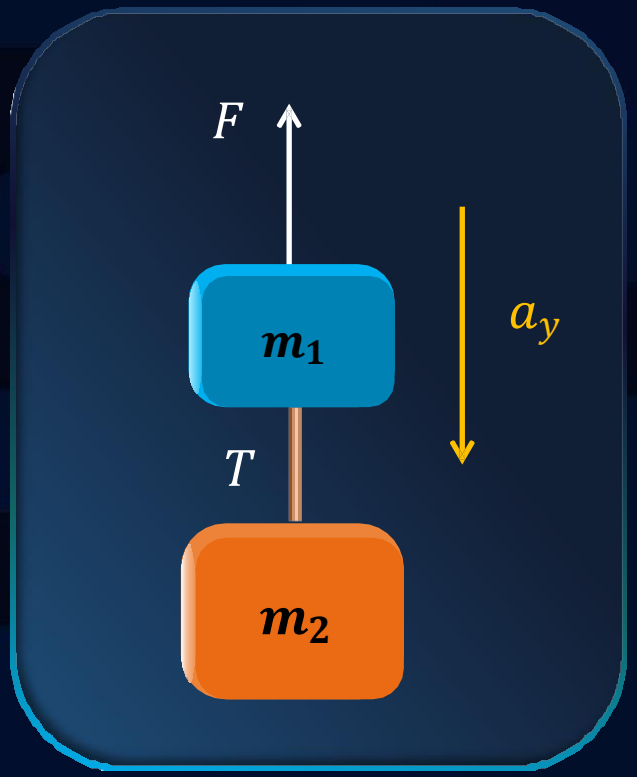
$$\Rightarrow F - m_1g - T = m_1a_y$$

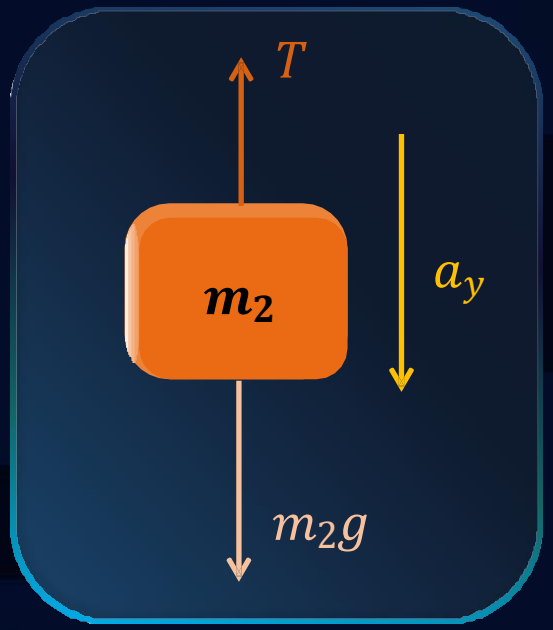
$$\Rightarrow F = (m_1 + m_2)(g + a_y)$$

$$F = (m_1 + m_2)g_{eff}$$

Vertical motion of the block

In Downward Direction:

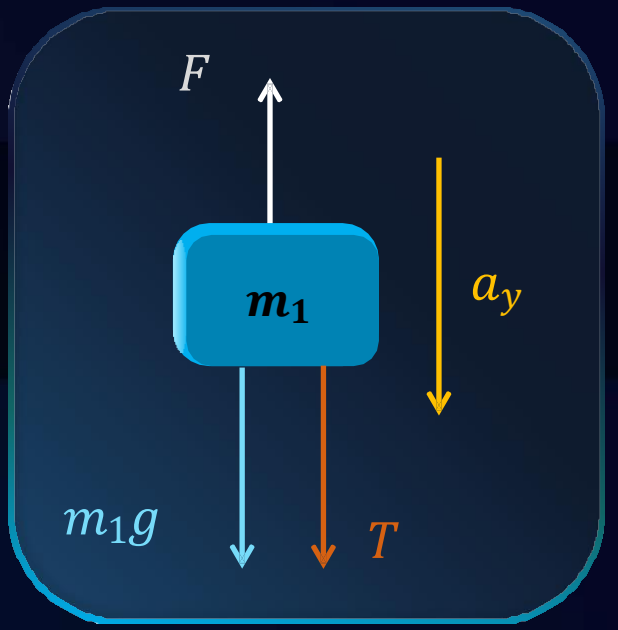




$$\Rightarrow m_2g - T = m_2a_y$$

$$\Rightarrow T = m_2(g - a_y)$$

$$T = m_2g_{eff}$$

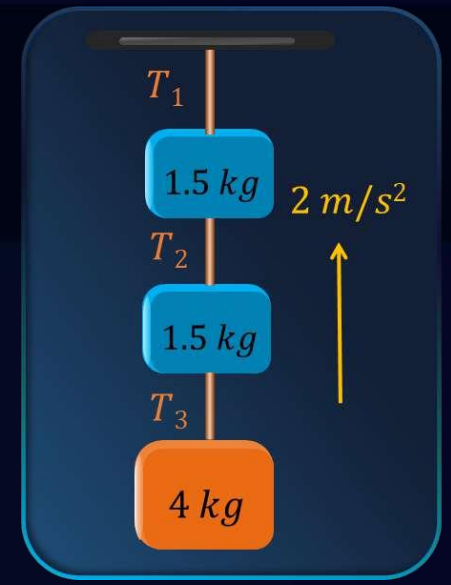


$$\Rightarrow m_1g + T - F = m_1a_y$$

$$\Rightarrow F = (m_1 + m_2)(g - a_y)$$

$$F = (m_1 + m_2)g_{eff}$$

Three blocks are suspended by a light string as shown in the figure below. What is the value of T_1, T_2 and T_3 if the whole system is moving up with an acceleration $2\ m/s^2$? (Take $g = 10\ m/s^2$)



Given: $m_1 = 1.5\ kg, m_2 = 1.5\ kg, m_3 = 4\ kg, a_y = 2\ m/s^2$

To find: $T_1, T_2, T_3 = ?$

Solution: $\Rightarrow T_3 = m_3(g + a)$



$$T_3 = 48\ N$$

$\Rightarrow T_2 = (m_1 + m_2)(a + g)$



$$T_2 = 66\ N$$

$\Rightarrow T_1 = (m_1 + m_2 + m_3)(g + a)$

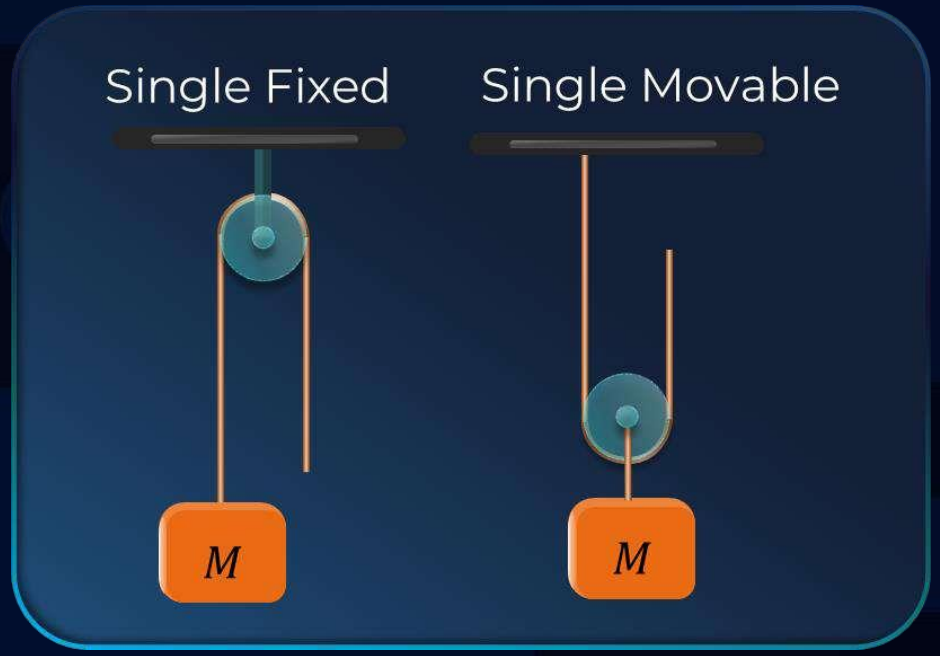


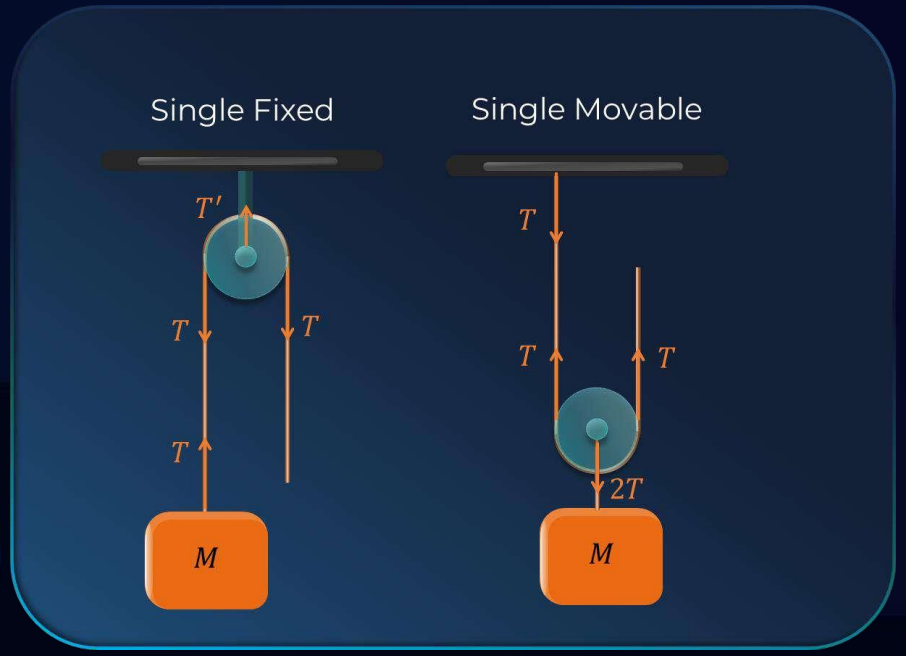
$$T_1 = 84\ N$$

Pulley

Assumption:

1. The rope is massless and inextensible.
2. The pulley is light, massless and frictionless.

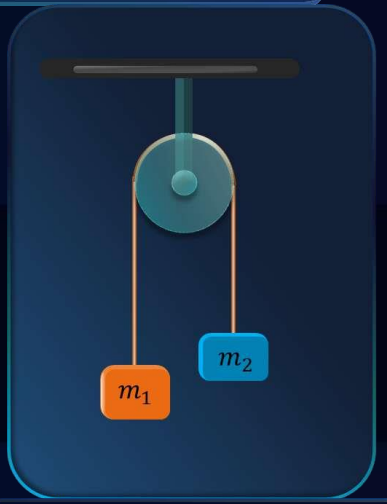


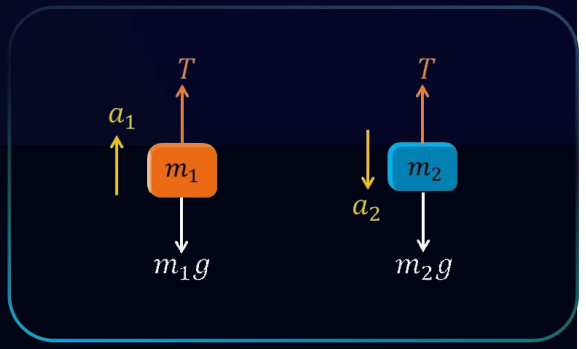


Steps to solve

Step I

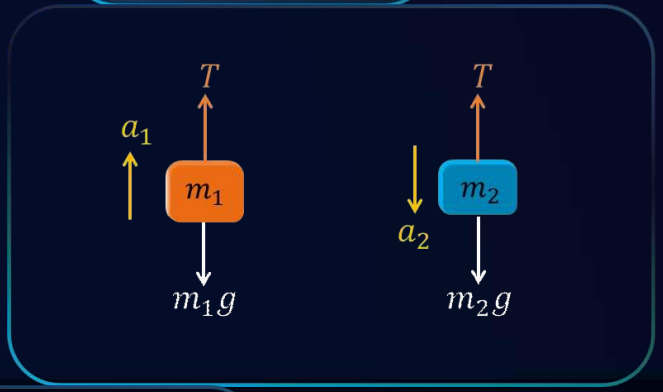
- Assume direction of accelerations.
- Draw FBD for all masses.
- If required resolve the forces along the axes.





Step II

Write equations using Newton's second law along each axis for each block.

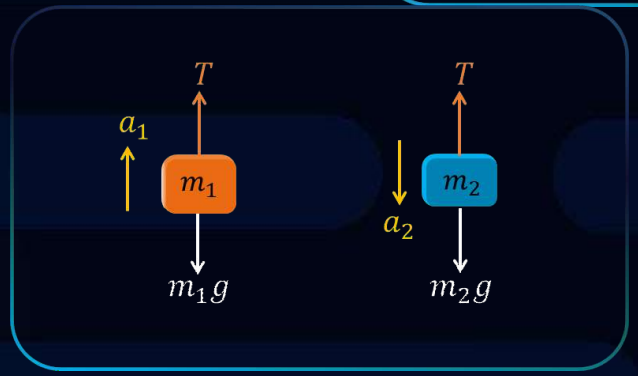


$$T - m_1 g = m_1 a_1$$

$$m_2 g - T = m_2 a_2$$

Step III

Solve the obtained equations.



$$T - m_1 g = m_1 a_1$$

$$a_1 = a_2$$

$$m_2 g - T = m_2 a_2$$

$$a = \left[\frac{m_2 - m_1}{m_1 + m_2}\right] g$$

$$T = \left[\frac{2 m_1 m_2}{m_1 + m_2}\right] g$$

After the blocks are released from rest, calculate the acceleration of the system and the amount of tension in the string.
(Take $g = 10\ m/s^2$)

Given: $m_1 = 2\ kg, m_2 = 3\ kg$

To find: $a = ?, T = ?$

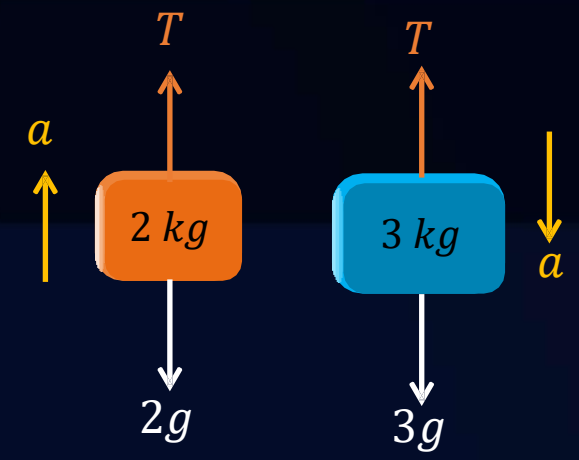
Solution: Method-I

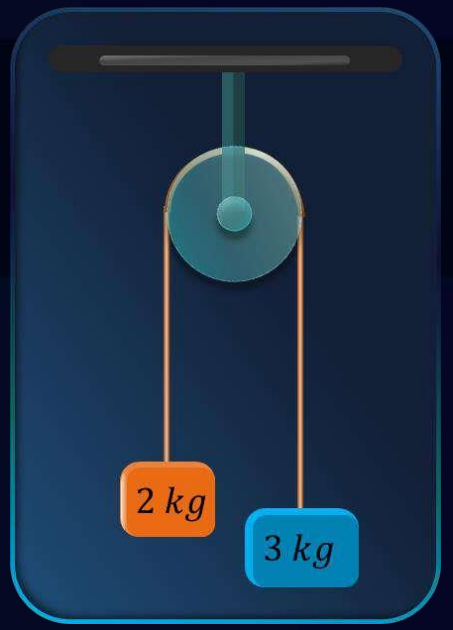
$\Rightarrow T - 2g = 2a$

$\Rightarrow 3g - T = 3a$

$a = 2\ m/s^2$

$T = 24\ N$





Method-II

$$\Rightarrow a = \frac{\text{Supporting force} - \text{Opposing force}}{\text{Total mass}} = \frac{30 - 20}{5} = 2\ m/s^2$$

$\Rightarrow T = m_2(g - a) = 3(10 - 2)\ N = 24\ N$

A block A of mass $7\ kg$ is placed on a frictionless table. A thread tied to it passes over a frictionless pulley and carries a body B of mass $3\ kg$ at the other end. The acceleration of the system is (Assume table to be smooth and take $g = 10\ m/s^2$)

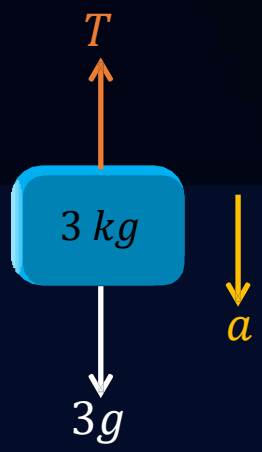
Given: $m_A = 7\ kg, m_B = 3\ kg$

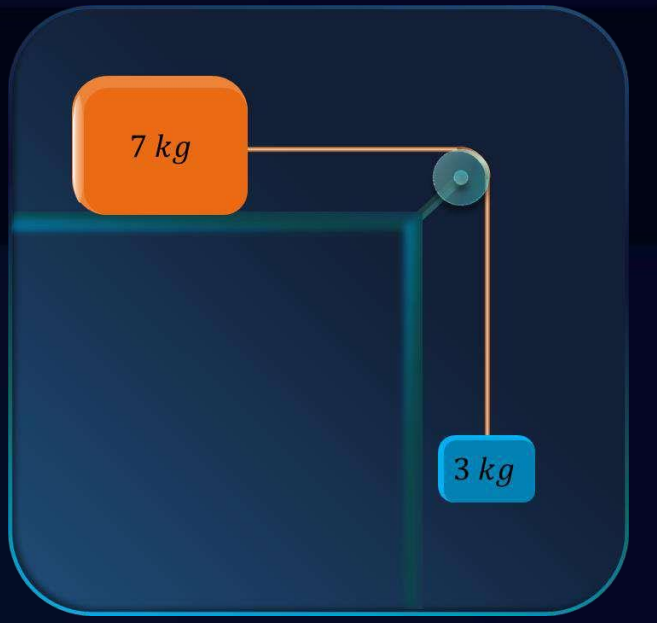
To find: $a = ?$

Solution: $\Rightarrow a = \dfrac{\text{supporting force} - \text{opposing force}}{\text{total mass}}$

$$\Rightarrow a = \frac{3g}{3+7} = \frac{3 \times 10}{10}\ m/s^2$$

$a = 3\ m/s^2$





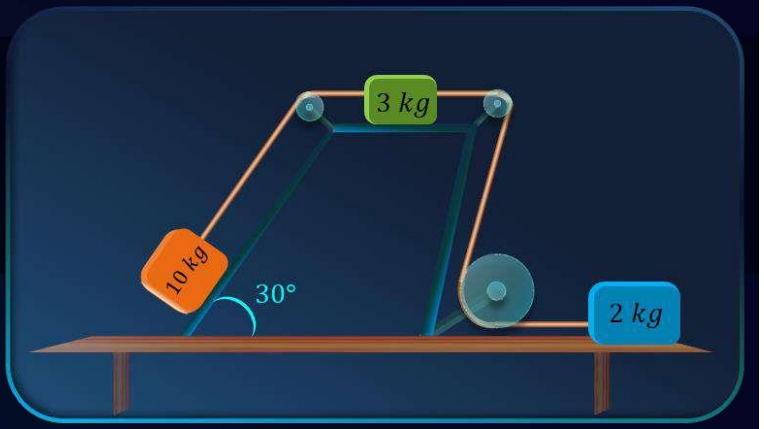
a. $100\ m/s^2$

b. $3\ m/s^2$

c. $10\ m/s^2$

d. $30\ m/s^2$

For the given system, evaluate the tension in each of the string which connects the blocks, respectively.
(Assume surface to be smooth and take $g = 10\ m/s^2$)



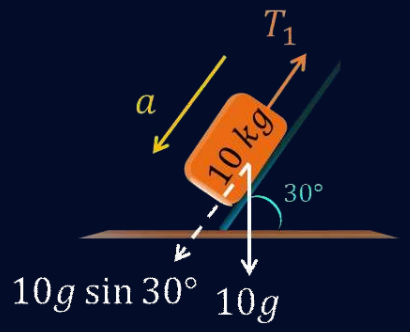
Given: $F = 100\ N\ kg, m_1 = 2\ kg, m_2 = 6\ kg, m_{AC} = 2\ kg, l_{AC} = 1\ m$

To find: $T_A, T_B, T_C = ?, a = ?$

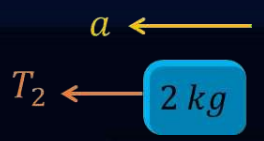
Solution: $$\Rightarrow a = \frac{\text{supporting force} - \text{opposing force}}{\text{total mass}}$$

$$\Rightarrow T_1 = 10\left(g\sin 30^0 - \frac{10}{3}\right) = \frac{50}{3}\ N$$

$$\Rightarrow T_2 = 2a = 2 \times \frac{10}{3} = \frac{20}{3}\ N$$







$$T_1 = \frac{50}{3}\ N, T_2 = \frac{20}{3}\ N$$

a. $\frac{50}{3}\ N, \frac{20}{3}\ N$

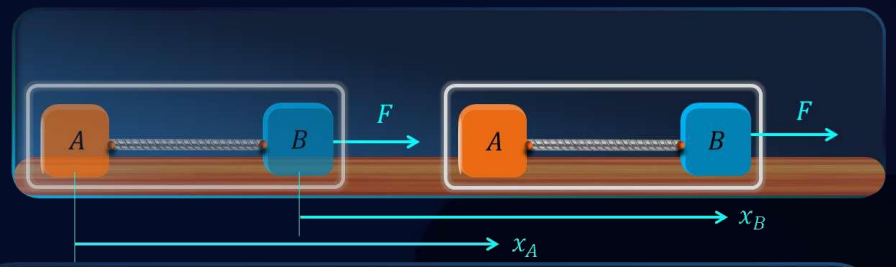
b. $\frac{25}{3}\ N, \frac{10}{3}\ N$

c. $\frac{50}{3}\ N, \frac{20}{7}\ N$

d. $50\ N, 20\ N$

Constrained Motion

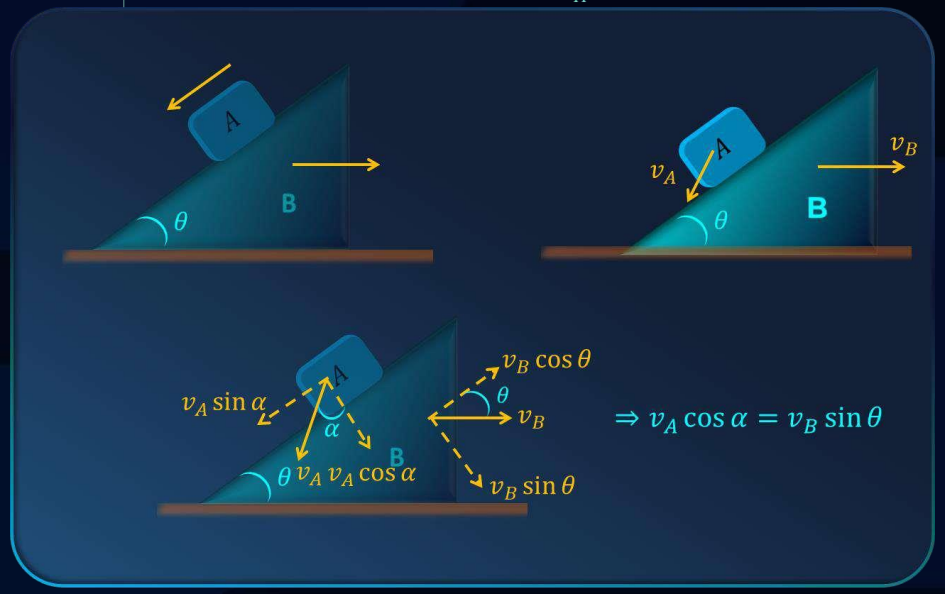
Rod-String Constrained Motion:



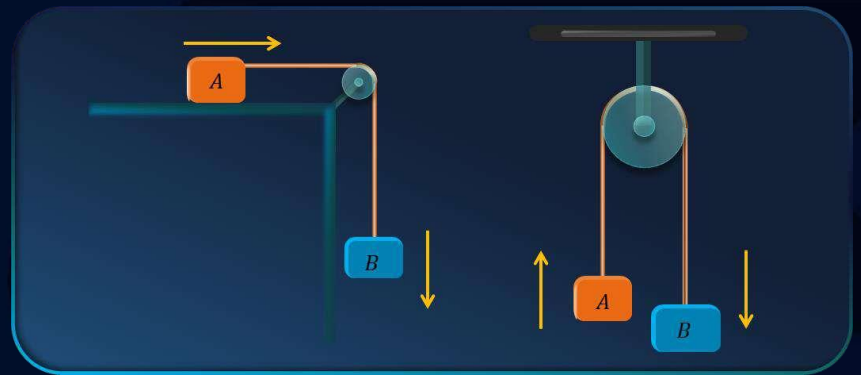
$$x_A = x_B \Rightarrow \frac{dx_A}{dt} = \frac{dx_B}{dt} \Rightarrow v_A = v_B$$

$$\Rightarrow \frac{dv_A}{dt} = \frac{dv_B}{dt} \Rightarrow a_A = a_B$$

Wedge-Block Constrained Motion:



Pulley-String Constrained Motion:



$$\Rightarrow x_A = x_B \Rightarrow \frac{dx_A}{dt} = \frac{dx_B}{dt} \Rightarrow v_A = v_B$$

$$\Rightarrow \frac{dv_A}{dt} = \frac{dv_B}{dt} \Rightarrow a_A = a_B$$

A rod moves vertically downwards on the surface of a wedge as shown in the figure. Find the relation between the velocity of the rod and that of the wedge at any instant.

Given : $v_A = 5\ ms^{-1}, \theta = 60^\circ$

To find : $v_B = ?$

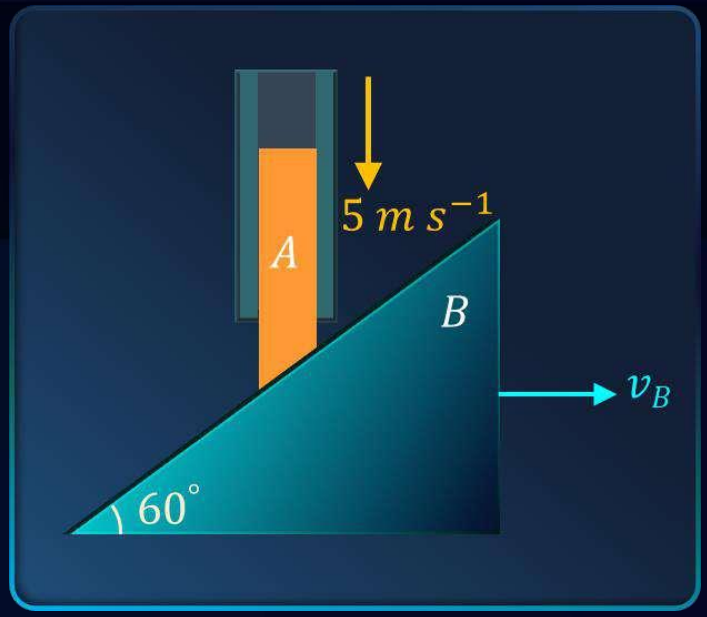
Solution : Using wedge constraint,

$$v_A \cos\theta = v_B \cos(90^\circ - \theta)$$

$$v_A \cos\theta = v_B \sin\theta$$

$$5 \times \frac{1}{2} = v_B \times \frac{\sqrt{3}}{2}$$

$$\therefore v_B = \frac{5}{\sqrt{3}}\ m\ s^{-1}$$



Weighing Machine

- Weight of a body is nearly constant near the earth's surface i.e., $W = mg$
- If the weighing machine measures the body's weight, then in both the cases it should give the same reading, but here we got two different readings. Why?
- Weighing machine measures the contact force exerted i.e., Normal reaction



Inertial Frame

An inertial frame of reference is a frame where Newton's laws hold true.

- Inertial frame is at rest or moves with constant velocity with respect to the assumed inertial reference frame i.e., $a_{frame} = 0$ (or) $v_{frame} = constant$

Car at rest



Car moving with constant velocity



Non-Inertial Frame

A non inertial frame of reference is a frame where Newton's laws does not hold true.

- Non-Inertial frame is accelerated with respect to the assumed inertial frame of reference i.e., $a_{frame} \neq 0$

Car moving with acceleration



Rotating disc





But, what about the person inside the lift. How will he write the equations?

Change in Normal Reaction



a

In lift frame



$$N = mg + ma$$

$$N = m(g + a)$$



a

In lift frame



$$N + ma = mg$$

$$N = m(g - a)$$

Pseudo force

- It is an imaginary force which is applied on objects in non-inertial frame of reference to satisfy Newton's laws of motion.

$$F_{pseudo} = -(m_{sys} \times a_{non-inertial\ frame})$$

- Acts opposite to the direction of acceleration of the frame
- Applied for non-inertial frame of reference
- No action-reaction pair.



In lift frame



$$N + mg = mg$$

$$N = m(g - g) = 0$$

The figure shows a pendulum suspended from the roof of a truck that has a constant acceleration a relative to the ground. Find the deflection of the pendulum from the vertical as observed from the ground frame and from the frame attached to the truck.

To find : $N = ?$

Solution : In ground frame,

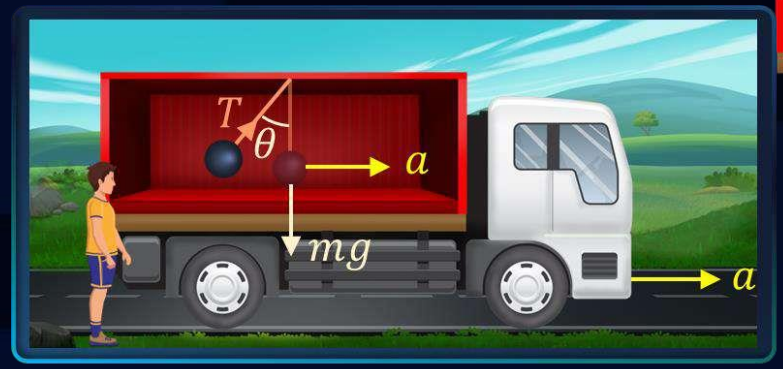
$$T = sin\theta = ma \quad \text{------} \ (1)$$

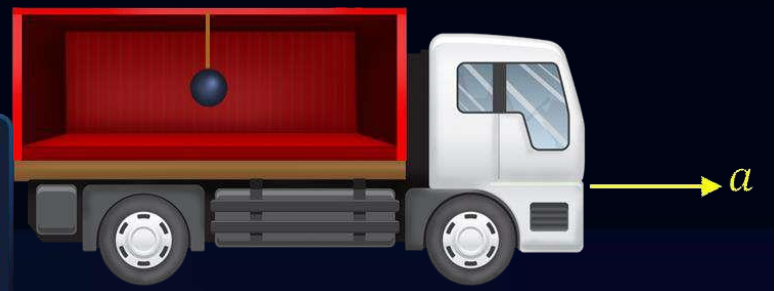
$$T = cos\theta = mg \quad \text{------} \ (2)$$

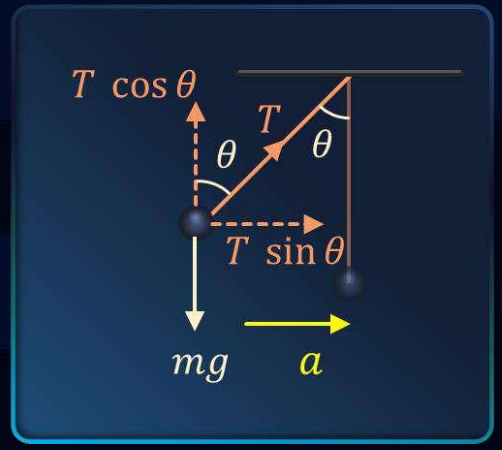
$From\ (1)\ and\ (2),$

$$\tan\theta = \frac{a}{g}$$

$$\theta = \tan^{-1}\left(\frac{a}{g}\right)$$



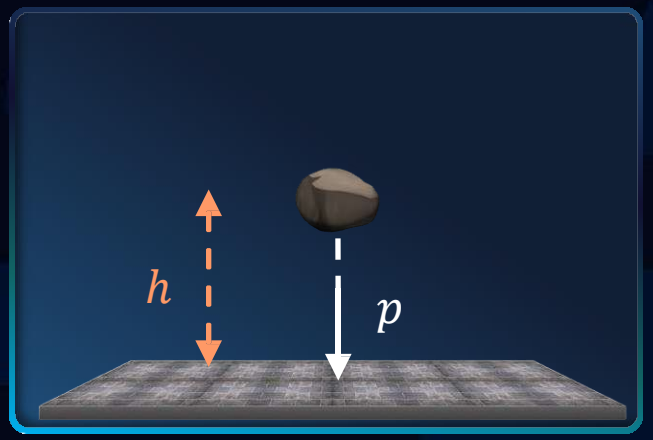




A stone is dropped from a height h. It hits the ground with a certain momentum p. If the same stone is dropped from a height $2h$, then momentum, when it hits the ground will change by

To find : $\Delta p = ?$

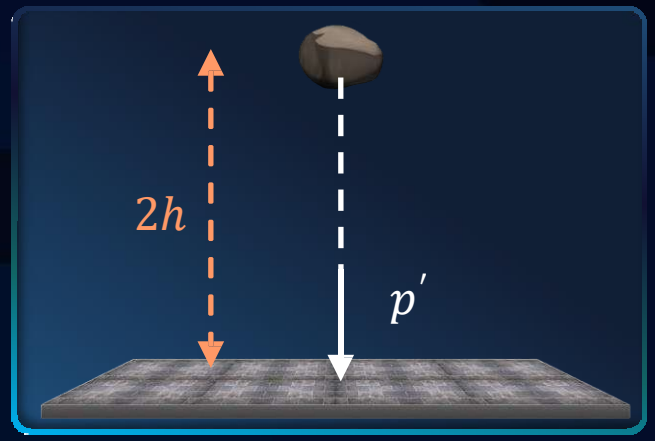
Solution :



$$v^2 = u^2 + 2gh$$

$$v_h^2 = 0 + 2gh$$

$$v_h = \sqrt{2gh}$$



$$v^2 = u^2 + 2gh$$

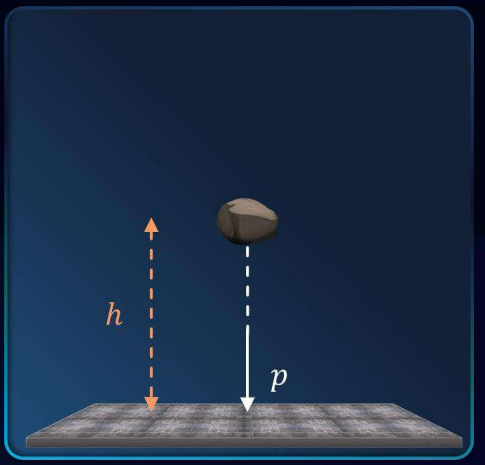
$$v_{2h}^2 = 0 + 2g(2h)$$

$$v_{2h} = \sqrt{4gh} = \sqrt{2}v_h$$

$$\Delta p = \frac{p' - p}{p} \times 100$$

$$\Delta p = \frac{m\sqrt{2}v_h - mv_h}{mv_h} \times 100$$

$$\Delta p = 0.41 \times 100 = 41\ \%$$



a. 68 %

b. 41 %

c. 200 %

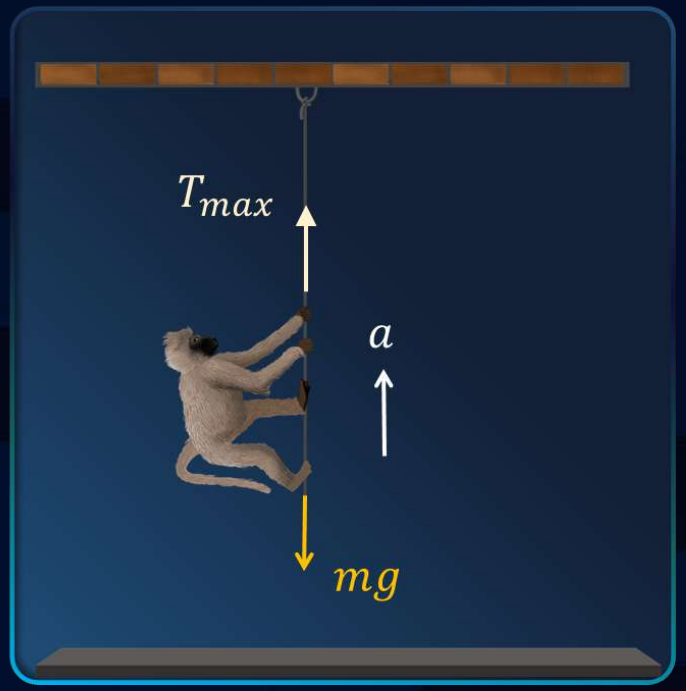
d. 100 %

A monkey of mass $20\ kg$ is holding a vertical rope. The rope will not break when a mass of $25\ kg$ is suspended from it but will break if the mass exceeds of $25\ kg$. What is the maximum acceleration with which the monkey can climb up along the rope ? $(g = 10\ ms^{-2})$

Given : $m = 20\ kg,\ m_{max} = 25\ kg$ and $g = 10\ m\ s^{-2}$

To find : $a = ?$

Solution :



As per question, $T_{max} < m_{max}g$

$$T_{max} = m_{max}g$$

$$T_{max} = 250\ N$$

$$T_{max} - mg = ma$$

$$250 - 200 = ma$$

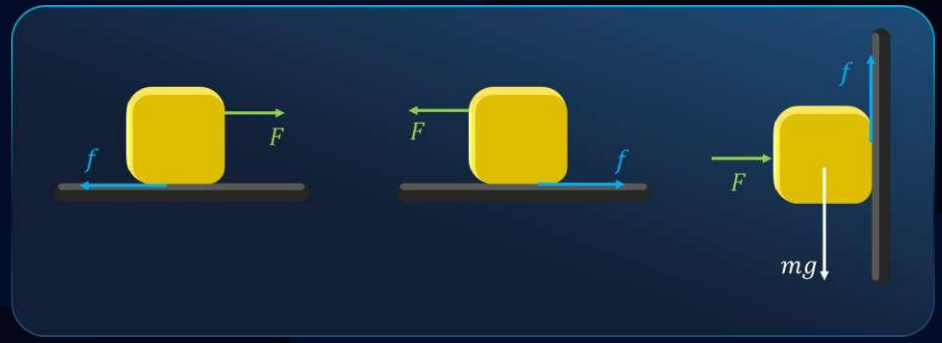
$$a = \frac{250 - 200}{20}$$

$$a = 2.5\ ms^{-2}$$



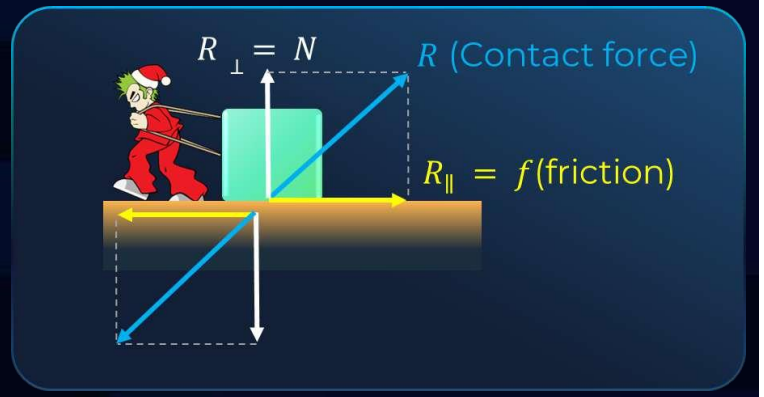
Friction : Introduction

- Friction is a force acting between two surfaces in contact, which opposes relative motion or tendency of relative motion between them.



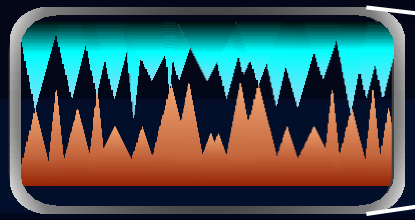
Contact Force : Recap

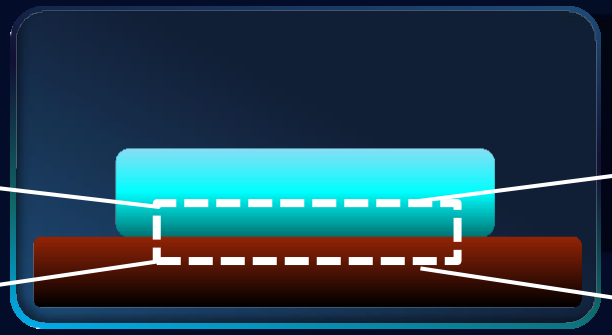
- The component of contact force that is parallel to the surface in contact is called, friction.
- Frictional force is electromagnetic in nature.



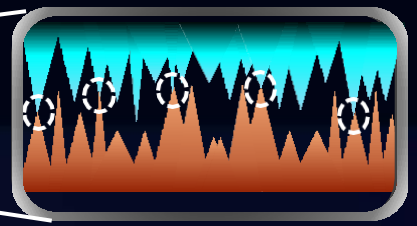
Friction : Microscopic view

Interlocking





Bonding

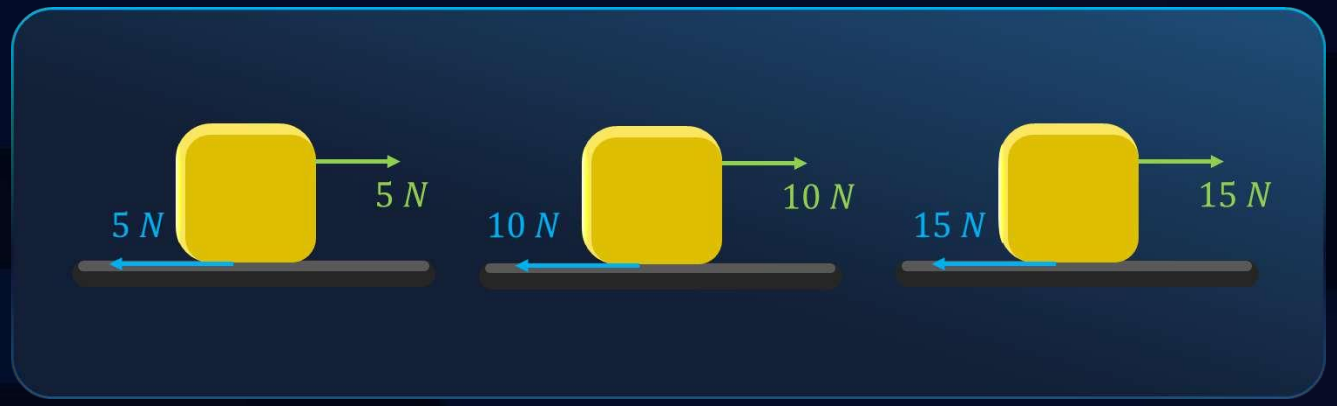


Interlocking between the irregularities in the surface gives rise to friction.

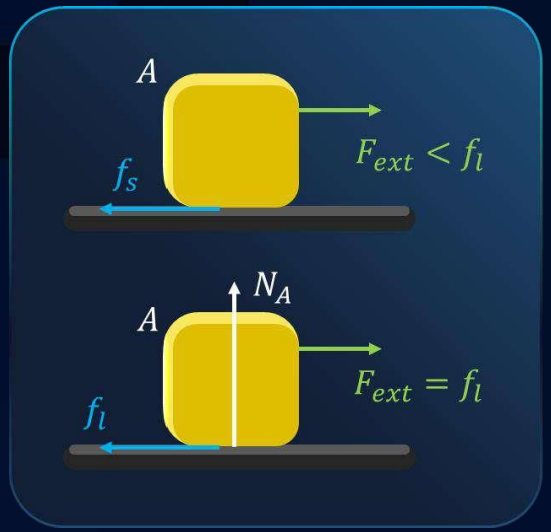
Temporary bonds are formed at the contact surface due to high pressure, this leads to friction.

Static Friction

- It is a variable resistive force which opposes tendency of relative motion.



Contact Force : Recap



- Maximum possible frictional force between two surfaces before sliding starts.
- If $F_{ext} > f_l$ then slipping/relative motion starts.

$$f_l \propto N_A$$

$$f_l = \mu_s N_A$$

- μ_s- coefficient of static friction

What will be value of limiting friction for a block of mass $5\ kg$ kept on a flat surface, if $\mu_s = 0.4$? If a force of $10\ N$ is applied on the block, will it move?(Take $g = 10\ m/s^2$)

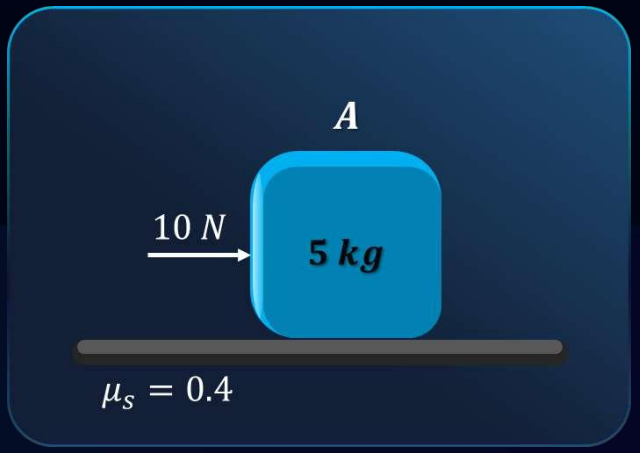
Solution :

$$N_A = mg = 5 \times 10 = 50\ N$$

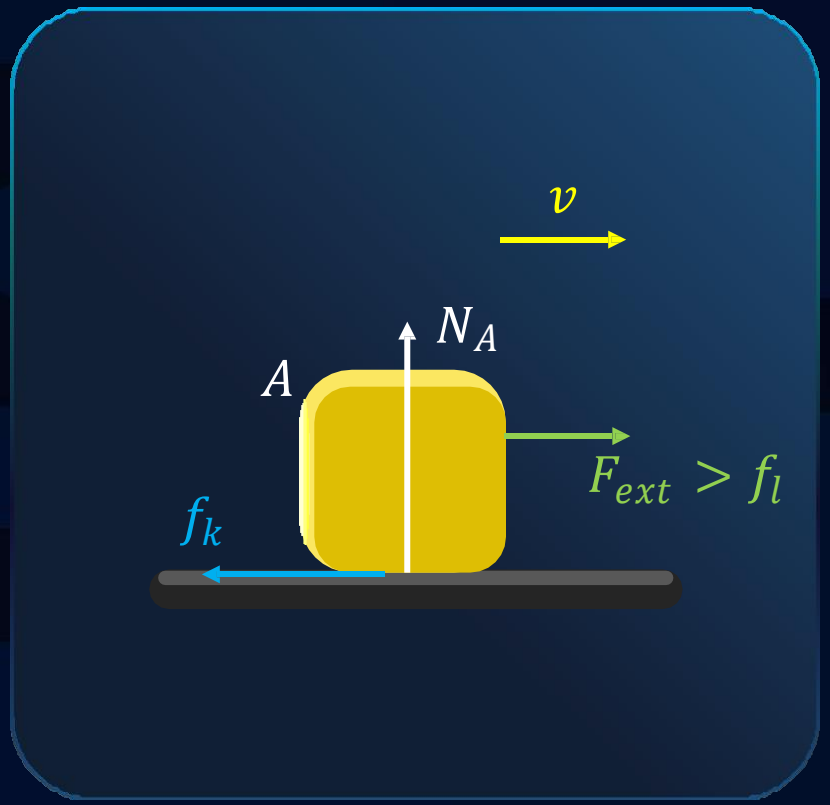
$$f_l = \mu_s N = 0.4 \times 50$$

$$f_l = 20\ N$$

$F < f_l \Rightarrow$ block remains stationary



Kinetic Friction



- If $F_{ext} > f_l$ then slipping/relative motion starts. The nature of friction switches to kinetic friction.
- Exists when there is relative motion between the surfaces.

$$f_k \propto N_A$$

$$f_k = \mu_k N_A$$

- μ_k- coefficient of kinetic friction

A force of $10\ N$ is applied on a block of mass $5\ kg$ as shown in the figure. Find the frictional force and the acceleration of the block.(Take $g = 10\ m/s^2$)

Given : $F = 10\ N, m_A = 5\ kg, \mu_s = 0.6, \mu_k = 0.5$

To find : f and a_A

Solution : $N_A = mg = 5 \times 10 = 50\ N$

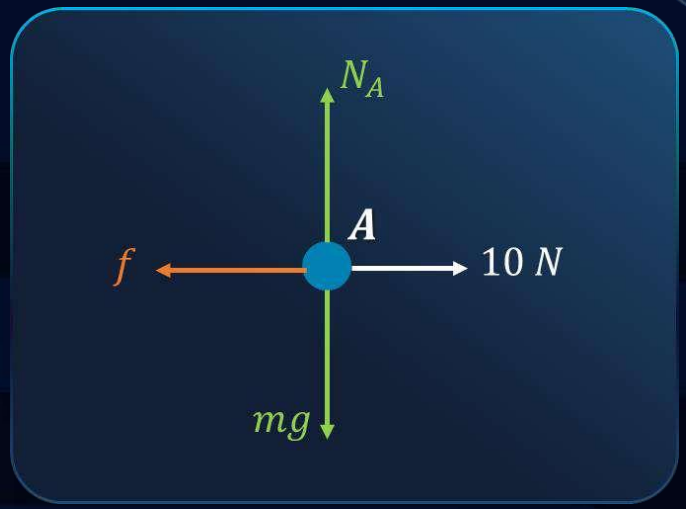
$f_l = \mu_s N = 0.6 \times 50$

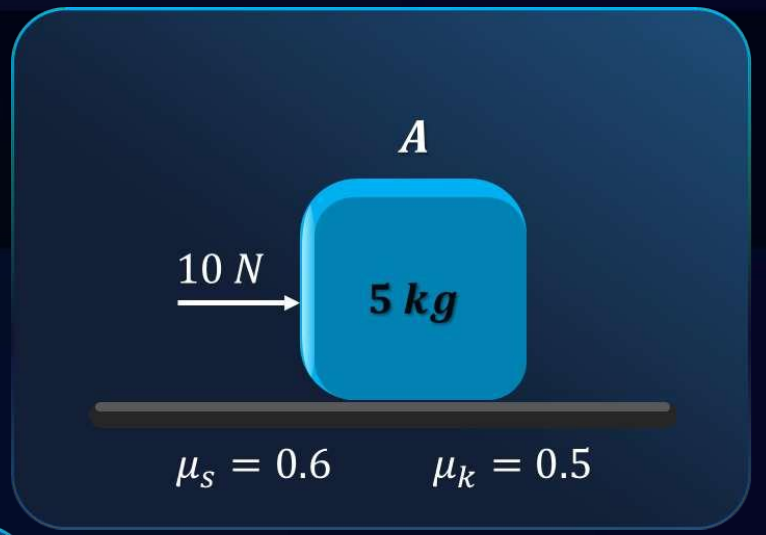
$f_l = 30\ N$

$F < f_l \Rightarrow$ static friction

Therefore, $f_s = F = 10\ N$

and $a_A = 0\ m/s^2$





a. $30\ N$ and $4\ m/s^2$

b. $10\ N$ and $4\ m/s^2$

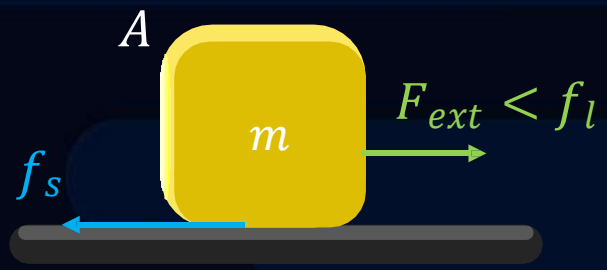
c. $10\ N$ and $0\ m/s^2$

d. $30\ N$ and $0\ m/s^2$

Tips for numerical

1

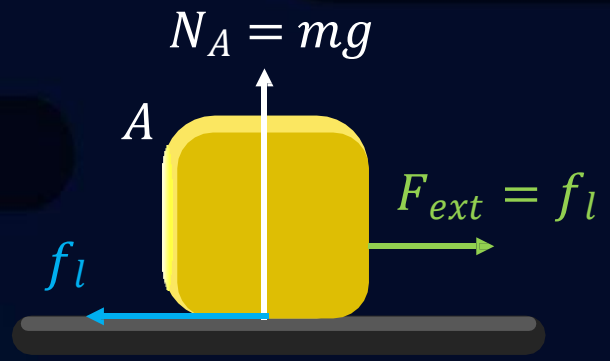
Rest



- $f_s = F_{ext}$
- $a = 0$

2

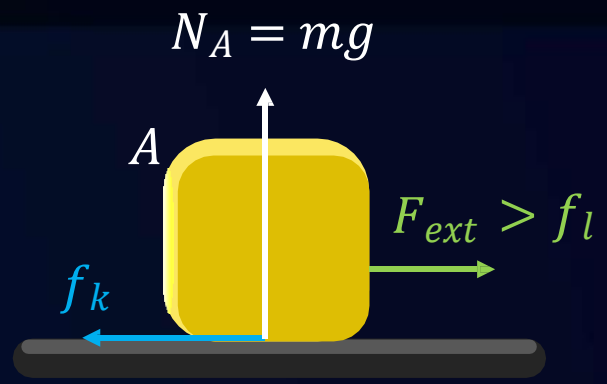
Just Slipping



- $f_l = \mu_s N_A$
- $a = 0$

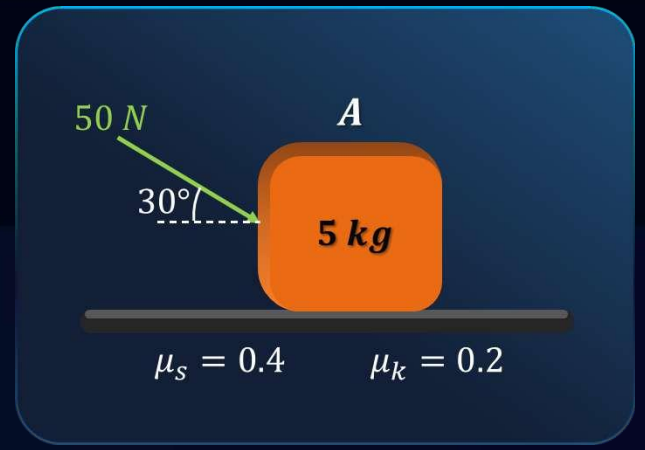
3

Sliding



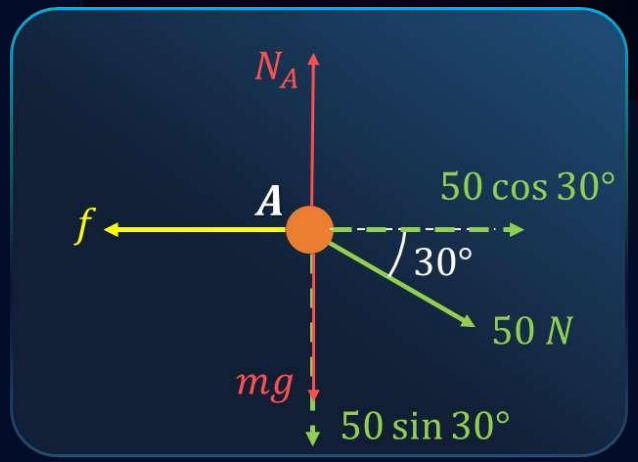
- $f_k = \mu_k N_A$
- $a = \frac{F_{ext} - f_k}{m}$

A force of $50\ N$ is applied on a block of mass $5\ kg$ as shown in the figure. Find the frictional force and the acceleration of the block.(Take $g = 10\ m/s^2$)



Given : $F = 50\ N, m_A = 5\ kg, \mu_s = 0.4, \mu_k = 0.2$

To find : f and a_A



Solution : $N_A = mg + 50 \sin 30°$

$$N_A = (5 \times 10) + \left(50 \times \frac{1}{2}\right) = 75\ N$$

$$f_l = \mu_s N_A = 0.4 \times 75 = 30\ N$$

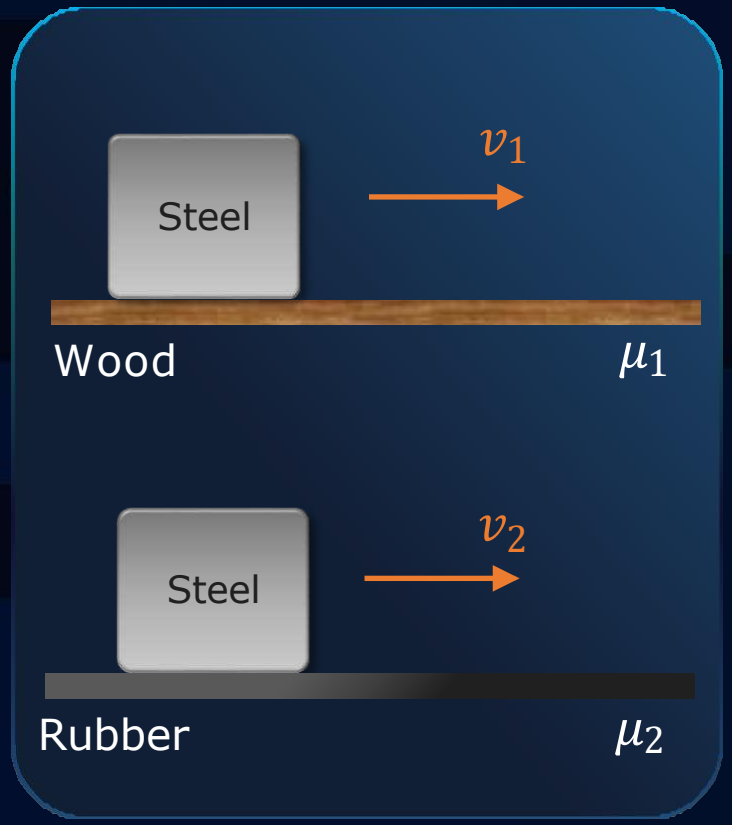
$$F_{ext} = 50 \cos 30° = 25\sqrt{3}\ N = 43.3\ N$$

$F_{ext} > f_l \Rightarrow f_k = \mu_k N_A = 0.2 \times 75$ $\quad f_k = 15\ N$

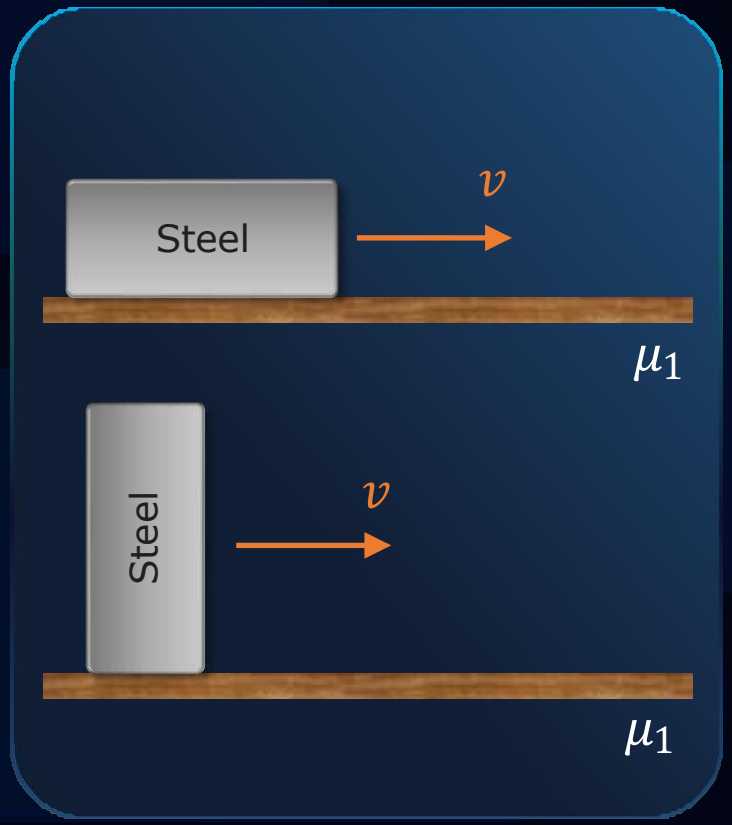
$$a_A = \frac{F_{ext} - f}{m} = \frac{43.3 - 15}{5}$$ $\quad a_A = 5.7\ m/s^2$

Properties of μ

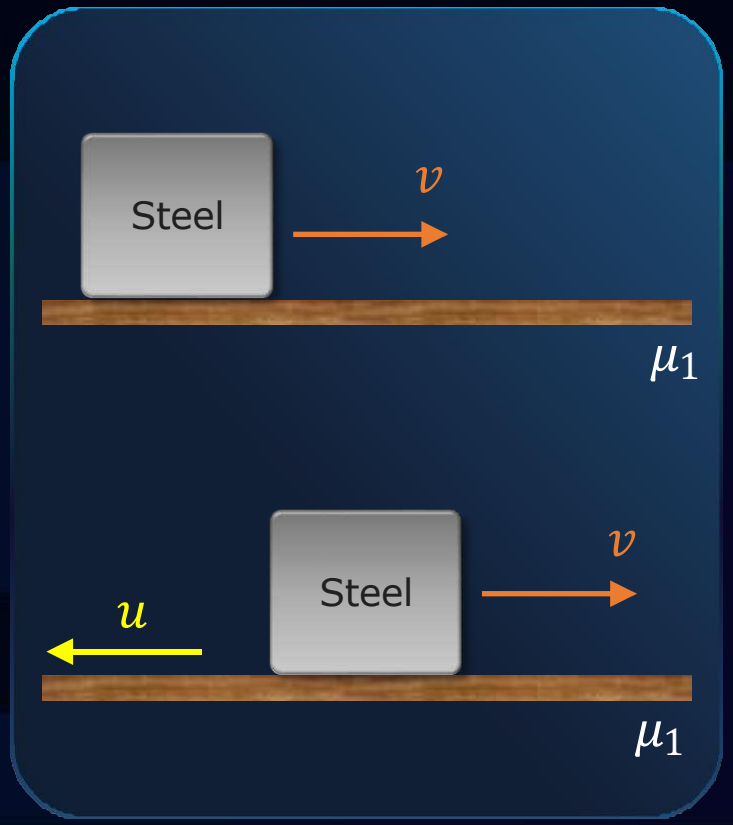
- μ is defined for a pair of material.



Depends on materials

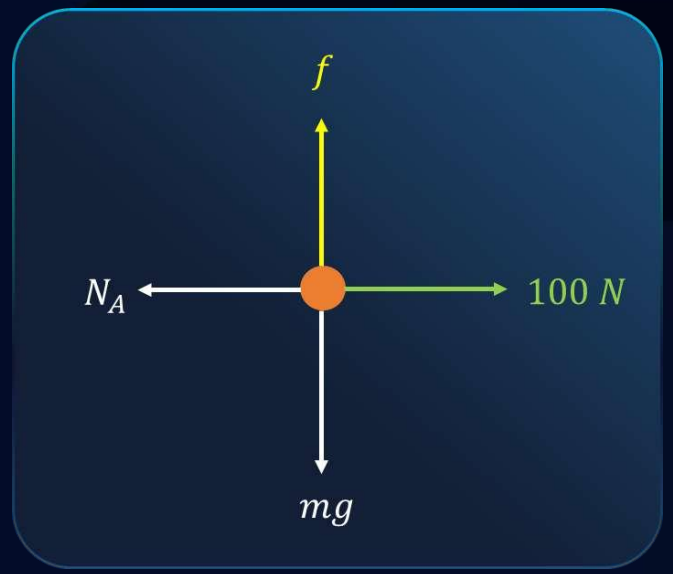


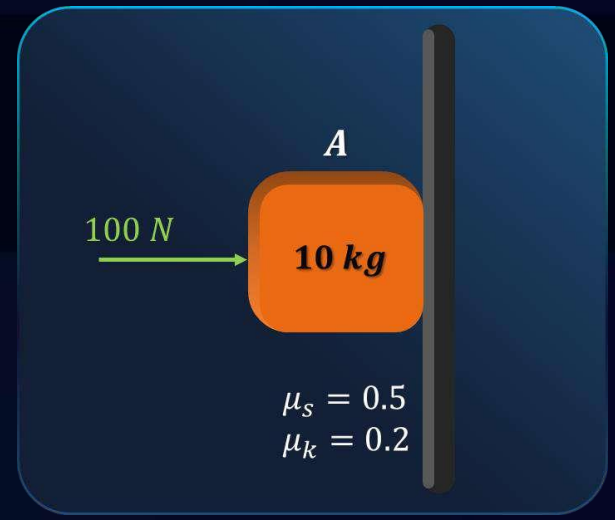
Independent of area



Independent of relative speed

A block of mass $10\ kg$ is pressed against a wall with $F = 100\ N$ as shown in the figure. Find the frictional force and the acceleration of the block. (Take $g = 10\ m/s^2$)





Given : $F = 100\ N, m_A = 10\ kg, \mu_s = 0.5, \mu_k = 0.2$

To find : f and a_A

Solution : $N_A = F = 100\ N$

$f_l = \mu_s N_A = 0.5 \times 100 = 50\ N$

$F_{y-ext} = mg = 10 \times 10 = 100\ N$

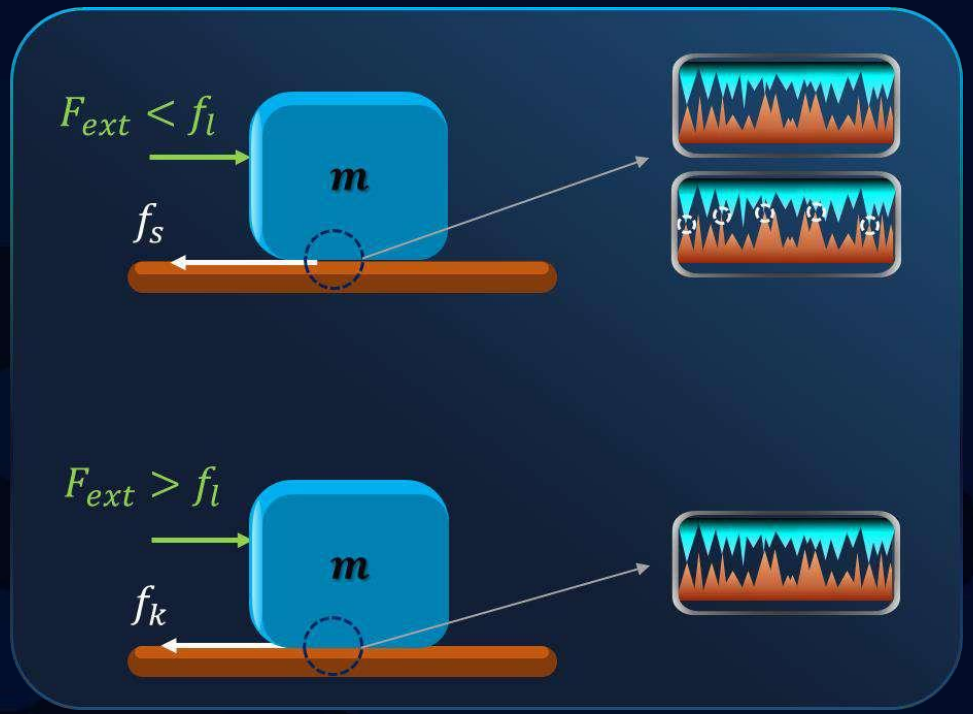
$F_y > f_l \Rightarrow f_k = \mu_k N_A = 0.2 \times 100$

$f_k = 20\ N$

$$a_A = \frac{F_y - f}{m_A} = \frac{100 - 20}{10}$$

$a_A = 8\ m/s^2$

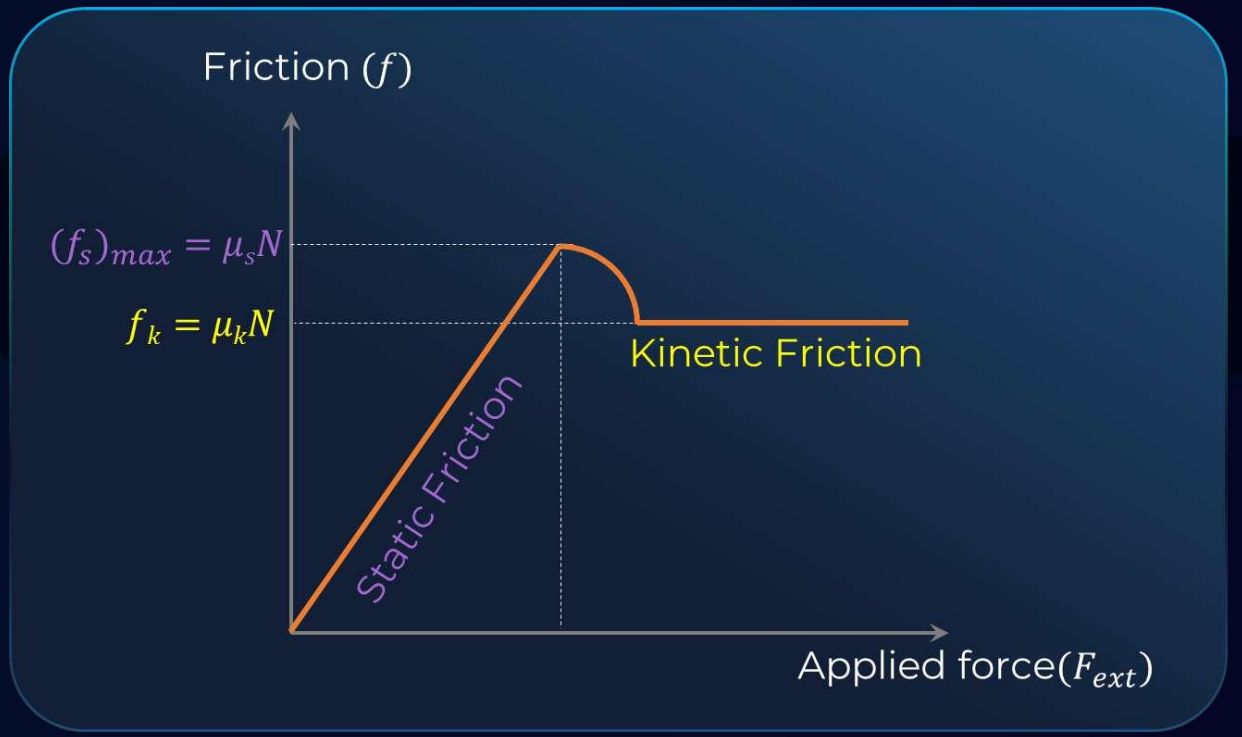
f_s and f_k



$$(f_s)_{\max} > f_k$$

$$\mu_s > \mu_k$$

Friction Graph



A block of mass $2\ kg$ is placed on a fixed inclined plane, as shown in the figure. The angle of inclination of the plane is $30°$. Find the minimum value of F, so that the block does not move.

Given : $m_A = 2\ kg, \mu = \mu_s = \mu_k = \frac{1}{2\sqrt{3}}$

To find : F

Solution : $N_A = 2g\cos 30° + F$

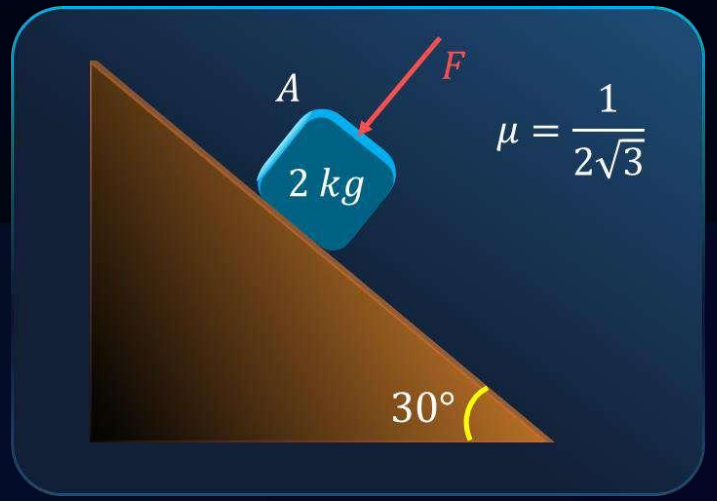
$$f_l = \mu_s N_A = \frac{1}{2\sqrt{3}}(20\cos 30° + F) \quad \text{----------} \ (1)$$

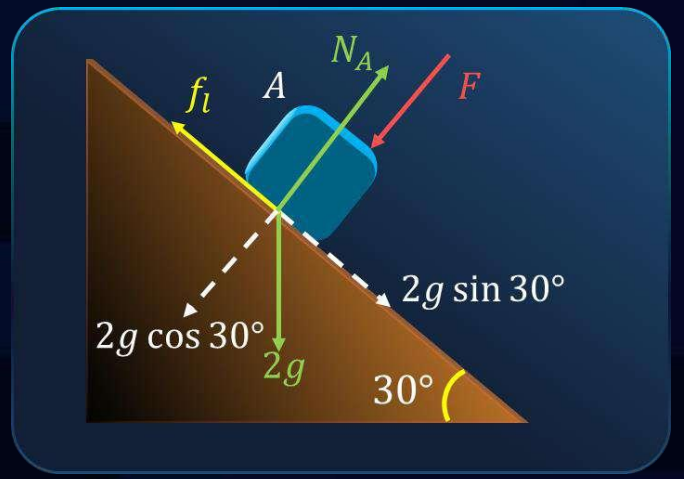
$$f_l = 2g\sin 30° = 10\ N \quad \text{----------} \ (2)$$

Subtituting $eq.2$ in $eq.1 \Rightarrow$

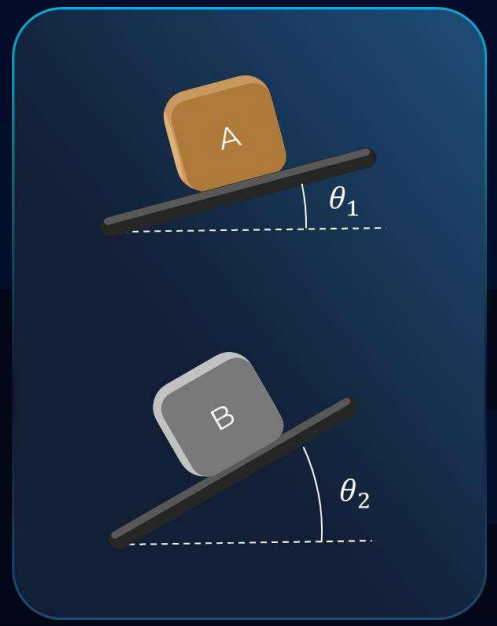
$$10 = \frac{1}{2\sqrt{3}}\left(20\frac{\sqrt{3}}{2} + F\right)$$

$$F = 10\sqrt{3}\ N$$

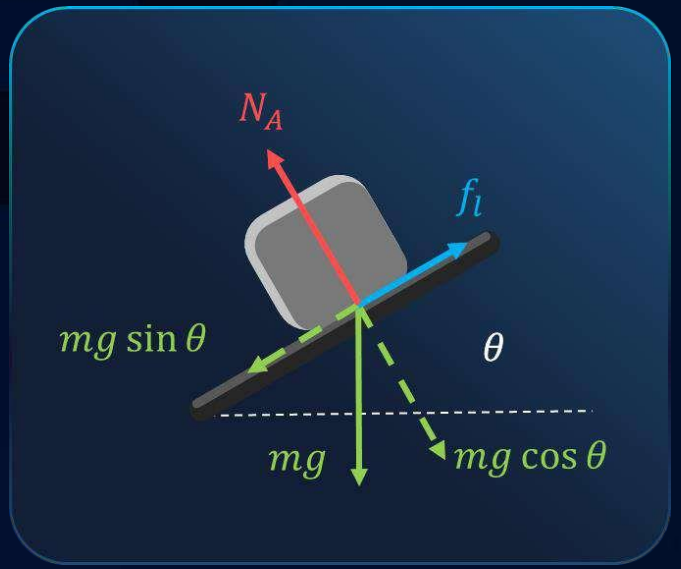




Angle of Repose



- The minimum angle (θ) made by the inclined plane with the horizontal such that the object placed on the inclined surface just begins to slide.
- θ is different for different materials in contact.



When angle of inclination = angle of repose The block just begins to slide.

$\Rightarrow f_l = mg \sin\theta$

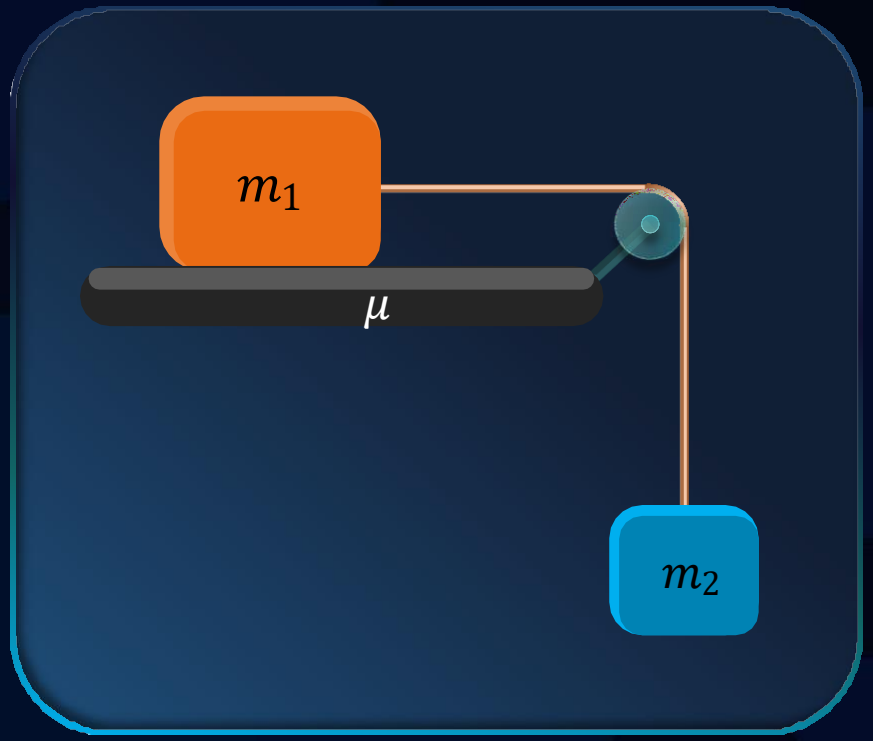
$\Rightarrow \mu_s N_A = mg \sin\theta$

$\Rightarrow \mu_s mg \cos\theta = mg \sin\theta$

$\Rightarrow \mu_s = \tan\theta$

$\Rightarrow \theta = \tan^{-1}(\mu_s)$

For the given arrangement, find the minimum value of mass m_1 for which the system remains in equilibrium. Assume pulley and string to be massless.



Solution :

Method I -

$$a = \frac{F_{ext} - f_l}{m_1 + m_2} \quad \text{----------} \ (1)$$

$$f_l = \mu m_1 g \quad \text{----------} \ (2)$$

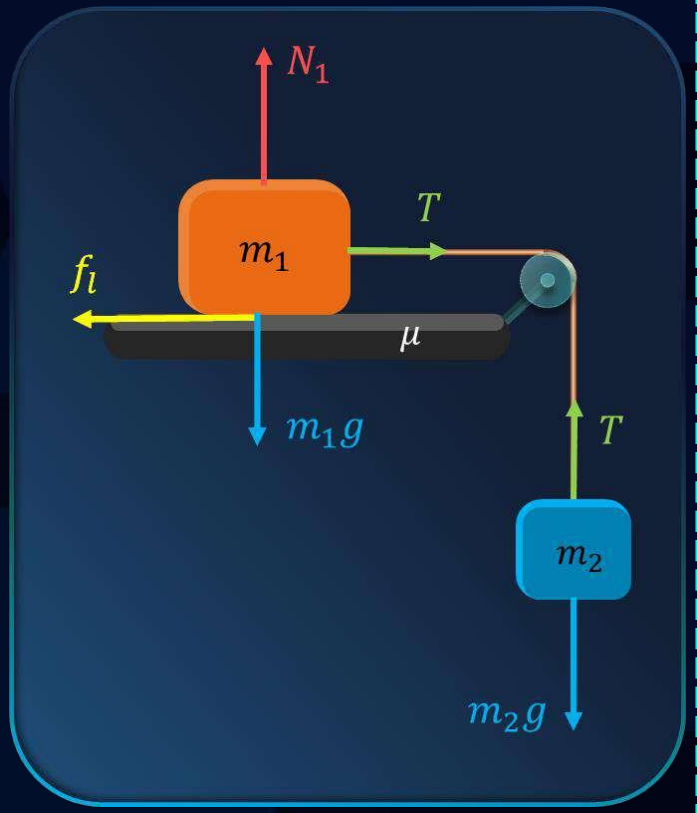
Subtituting $eq.2$ in $eq.1 \Rightarrow$

$$a = \frac{m_2 g - \mu m_1 g}{m_1 + m_2}$$

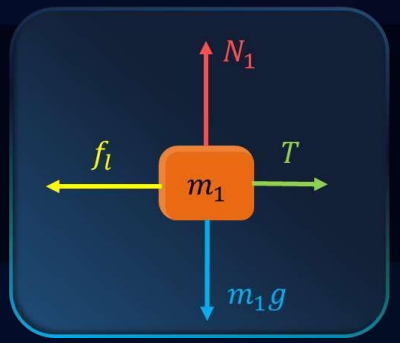
At equilibrium, $a = 0$

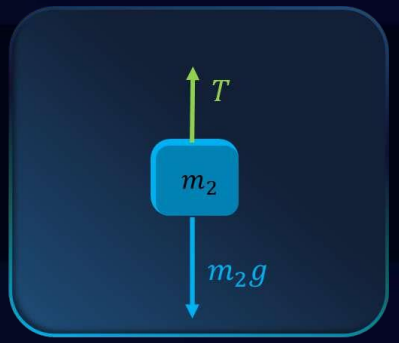
$$\Rightarrow \mu m_1 g = m_2 g$$

$$m_1 = \frac{m_2}{\mu}$$



Solution : Method II -





$$T = f_l = \mu m_1 g \quad \text{----------} \ (1)$$

$$T = m_2 g \quad \text{----------} \ (2)$$

Equating $eq.2$ and $eq.1 \Rightarrow \quad \mu m_1 g = m_2 g$

$$m_1 = \frac{m_2}{\mu}$$

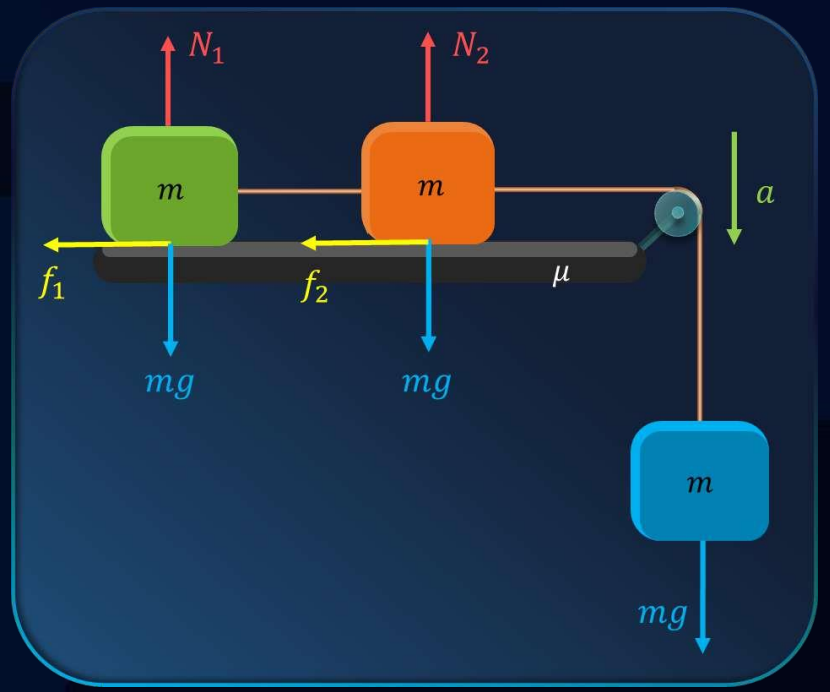
For the given system, evaluate the acceleration of the system.

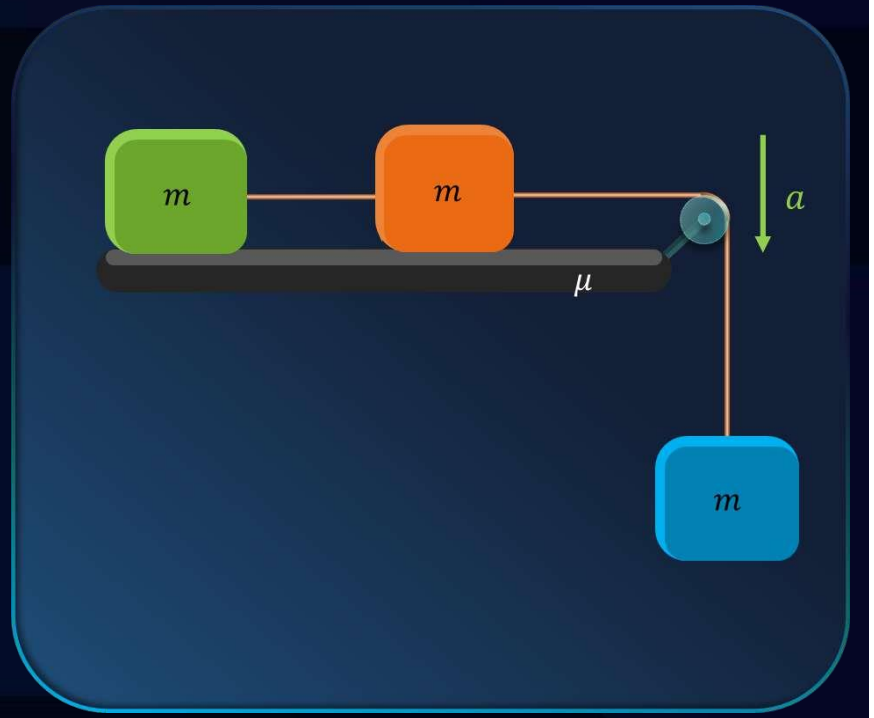
Solution :

$$a = \frac{F_{ext} - f}{3m} = \frac{mg - (f_1 + f_2)}{3m}$$

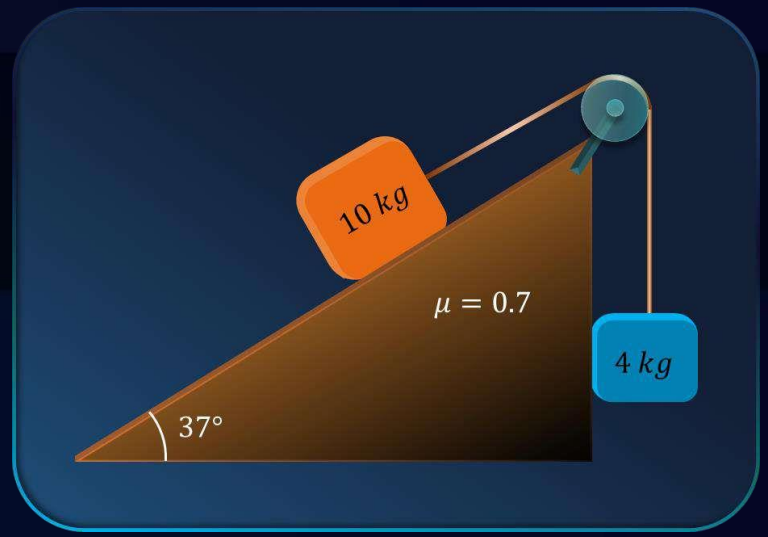
$$a = \frac{mg - 2\mu mg}{3m}$$

$$a = \frac{g(1 - 2\mu)}{3}$$





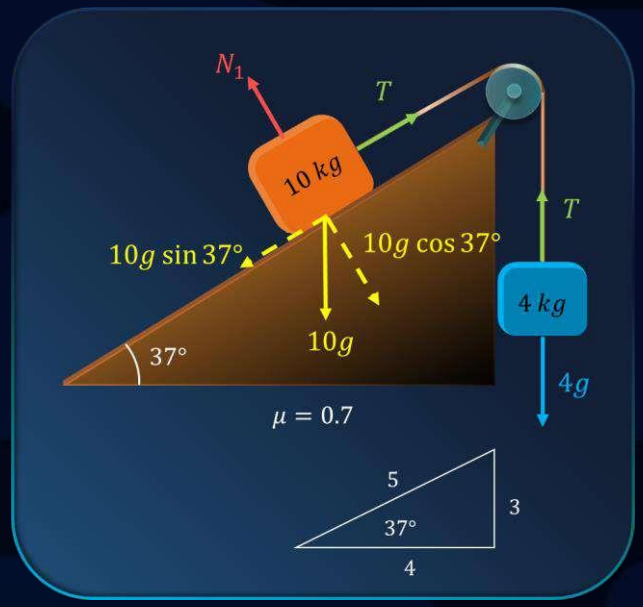
For the arrangement shown in the figure, if the system is in equilibrium, the value of frictional force on $10\ kg$ block is -



Given : $m_1 = 10\ kg, m_2 = 4\ kg$ and $\mu = 0.7$

To find : f

Solution : $T = 4g = 40\ N$



For block of $10\ kg \Rightarrow$

$$10g \sin 37° = 100 \times \frac{3}{5} = 60\ N$$

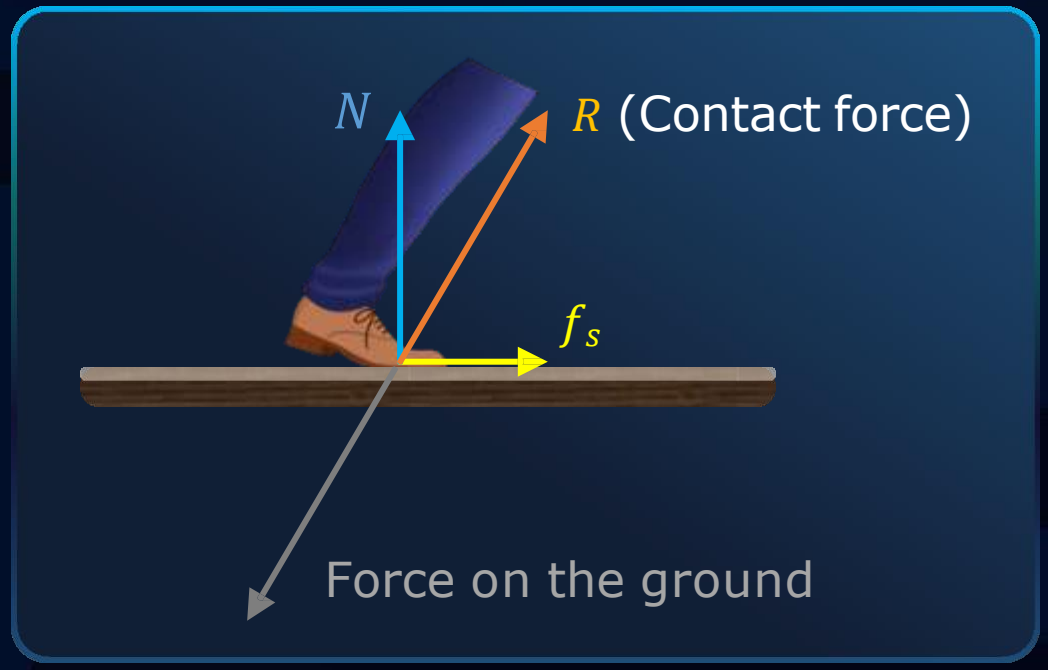
The system is at equilibrium

$$f = 10g \sin 37° - T$$

$$f = 20\ N$$

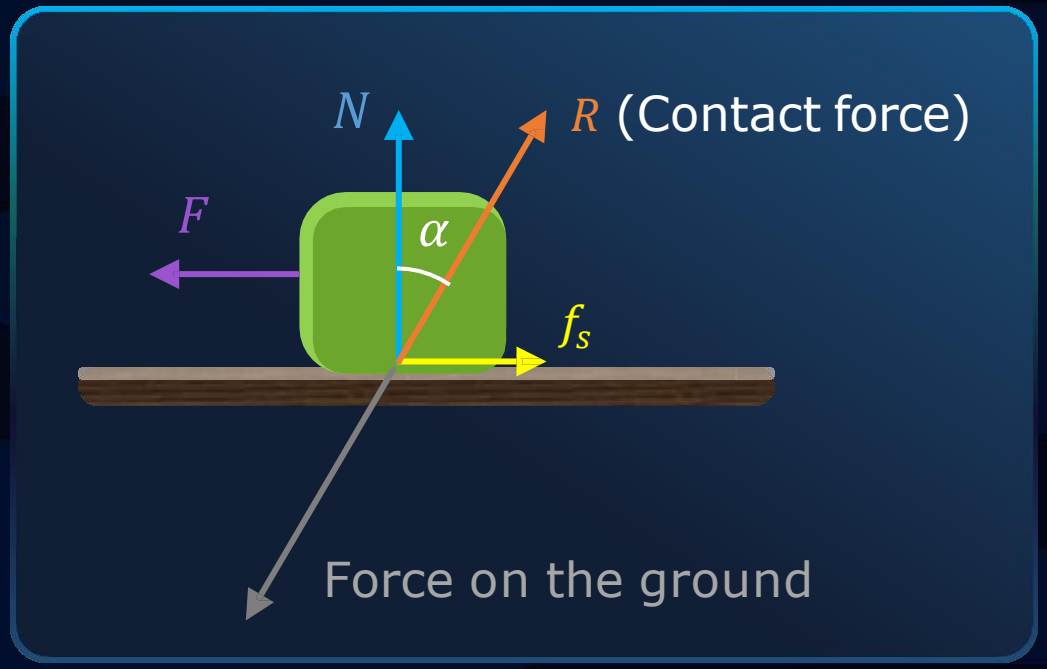
Role of Friction in Walking

- While walking, the foot presses the ground at an angle in the backward direction. The ground, in turn, exerts a reaction force R on the foot.
- The frictional force prevents us from slipping.



Angle of Friction

- The angle made by the resultant of normal reaction and limiting friction, with the normal reaction is called the angle of friction (α).



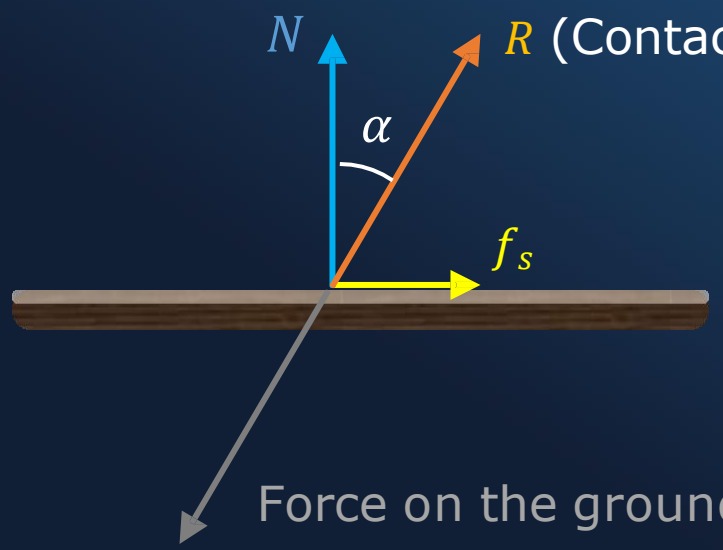
$$\tan\alpha = \frac{f_l}{N}$$

$$\tan\alpha = \frac{\mu_s N}{N} = \mu_s$$

$$\alpha = \tan^{-1}\mu$$

The coefficient of friction between two surfaces is 0.2. The angle of friction is

Solution :



$$\alpha = \tan^{-1}\mu$$

$$\tan\alpha = \mu = 0.2$$

$$\tan\alpha = \frac{2}{10} = \frac{1}{5}$$

$$\cot\alpha = 5$$

$$\alpha = \cot^{-1} 5$$

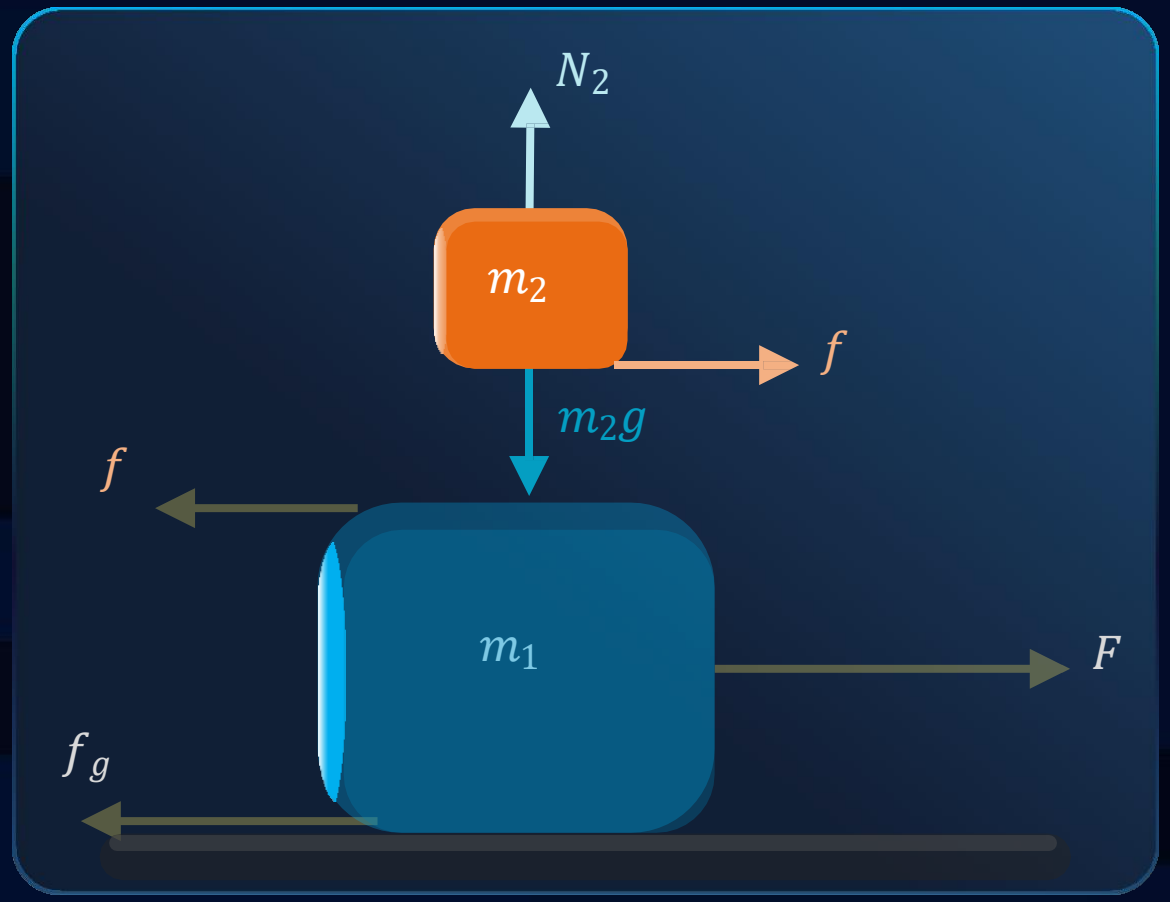
a. $\sin^{-1} 0.2$

b. $\cot^{-1} 5$

c. $\tan^{-1} 0.1$

d. $\cos^{-1} 0.2$

Block on block



$$f_l = \mu_2 N_2$$

$$f_l = \mu_2 m_2 g$$

$$m_2 a_{max} = \mu_2 m_2 g$$

$a_{max} = \mu_2 g$

$a \leq a_{max}$

Both blocks move together with common acceleration, a_s and can be assumed as one system.

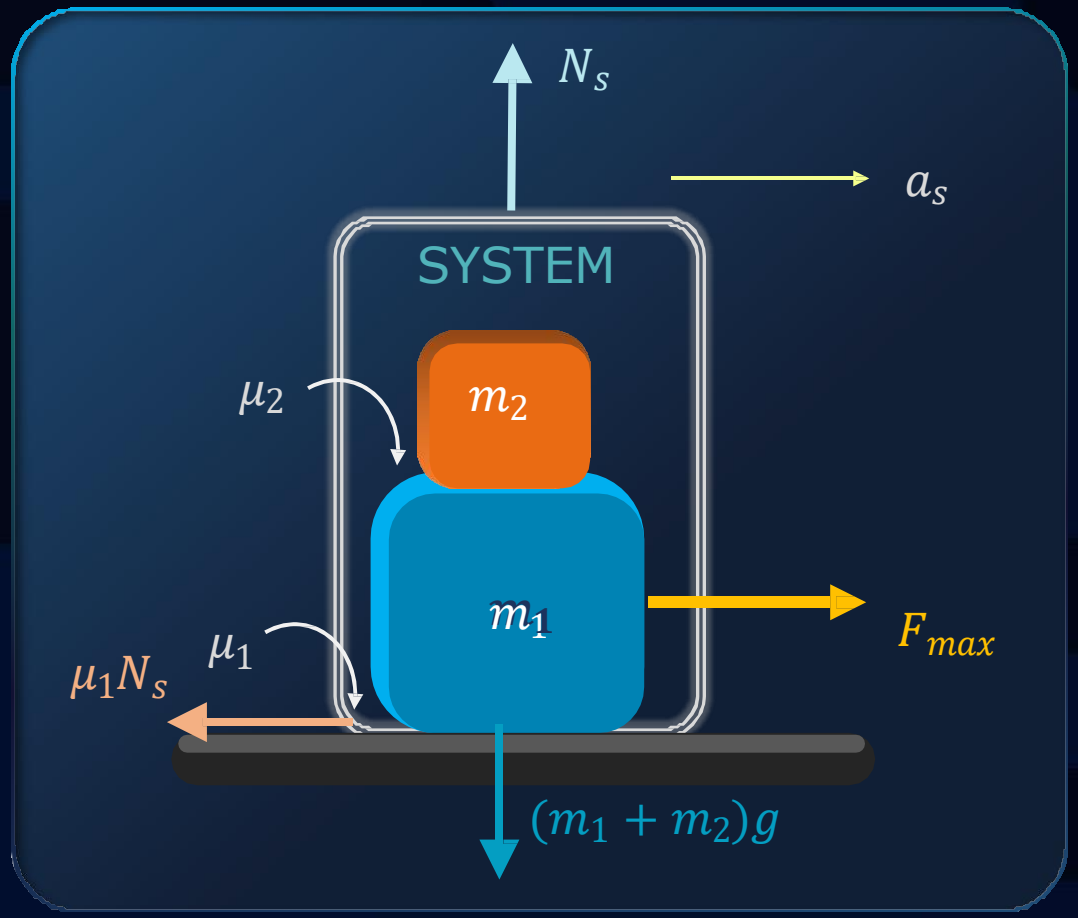
$a > a_{max}$

Each block move with different acceleration

Force on the bottom block

What is the maximum force applied on the block m_1, so that both the blocks move together with a common acceleration?

Assumption : $a \leq a_{max} \Rightarrow$ System move with common acceleration, $a_s = \mu_2 g$



$$F_{max} - \mu_1 N_s = M a_s$$

$$F_{max} = M a_s + \mu_1 N_s$$

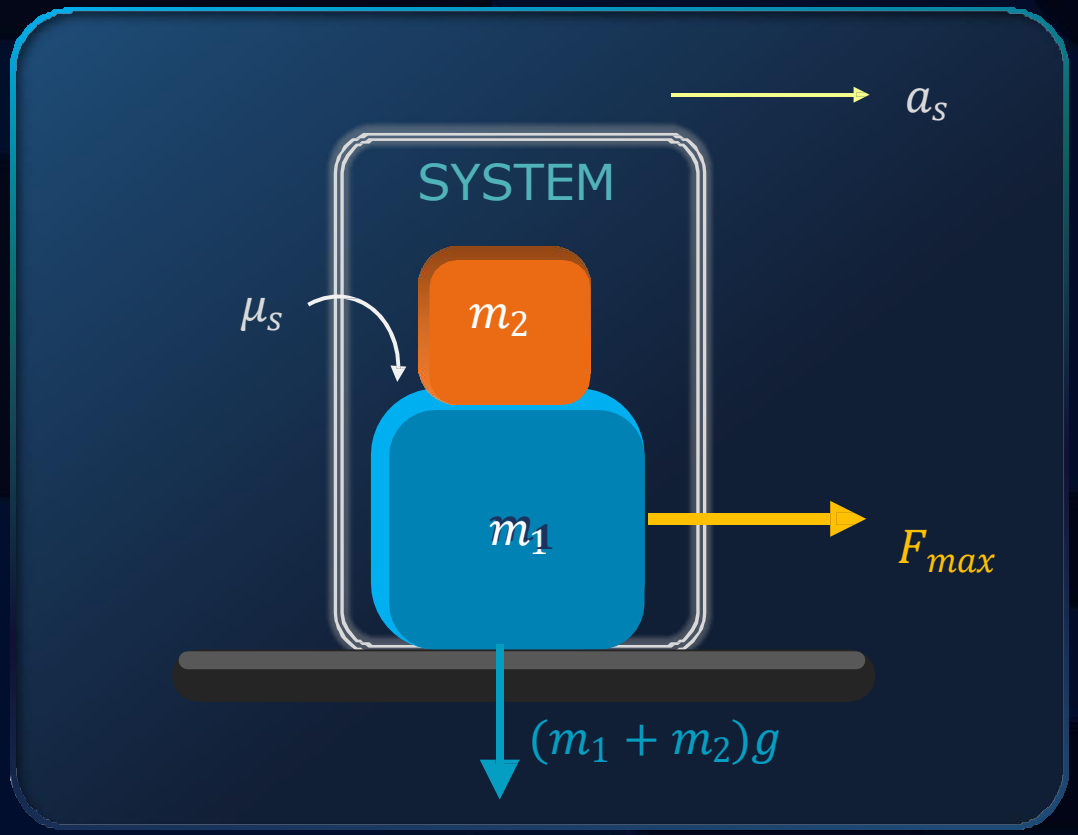
$$F_{max} = (m_1 + m_2)\mu_2 g + \mu_1 (m_1 + m_2) g$$

$$F_{max} = (m_1 + m_2)(\mu_1 + \mu_2) g$$

Force on the bottom block

What is the maximum force applied on the block m_1, so that both the blocks move together with a common acceleration?

Case : When the ground is smooth $\Rightarrow \mu_1 = 0, \mu_2 = \mu_s$

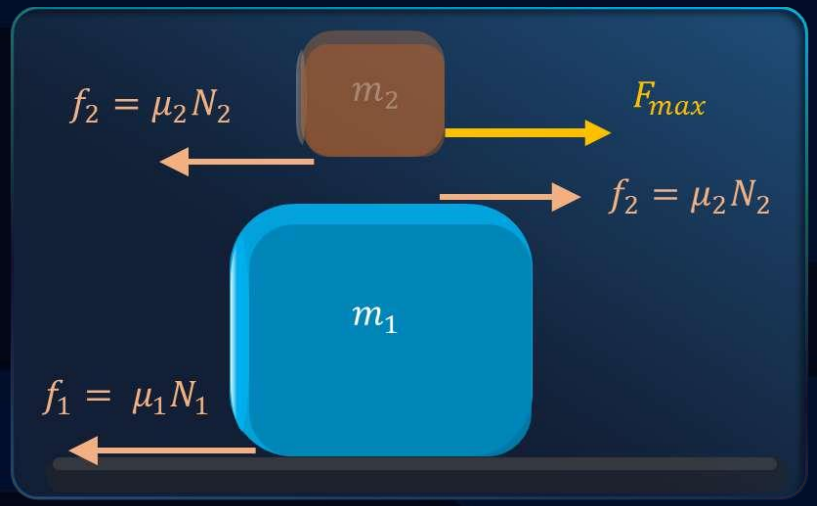


$$F_{max} = (m_1 + m_2)(\mu_1 + \mu_2)g$$

$$F_{max} = (m_1 + m_2)\mu_s g$$

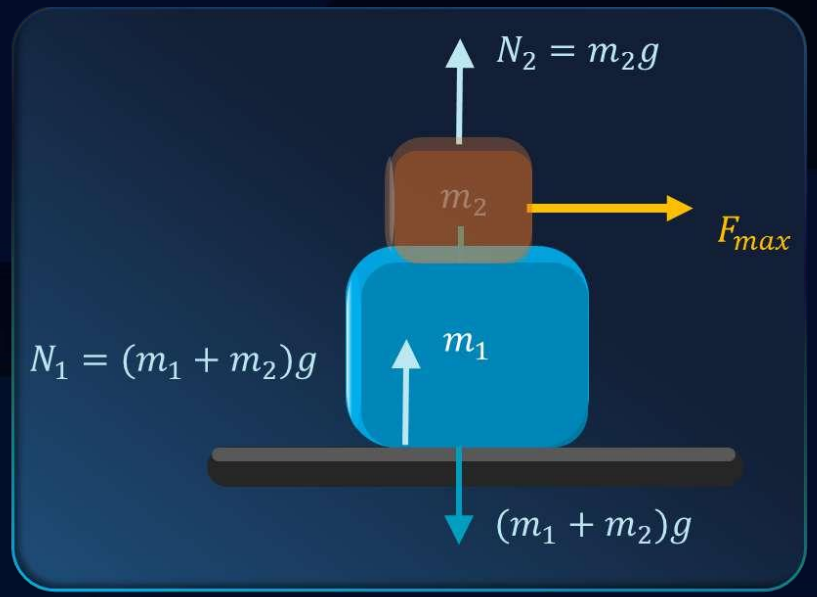
Force on the top block

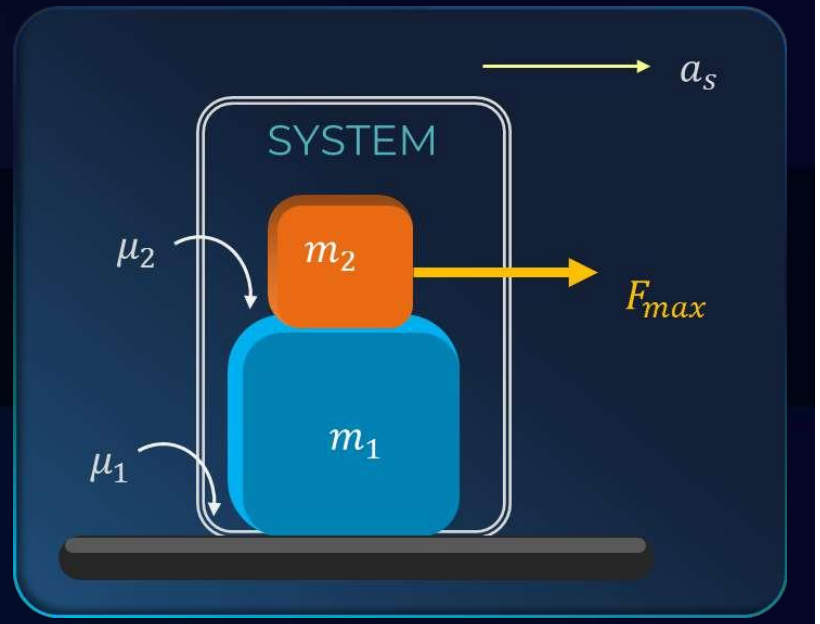
What is the maximum force applied on the block m_2, so that both the blocks move together with a common acceleration?



$$f_2 - f_1 = m_1 a_s$$

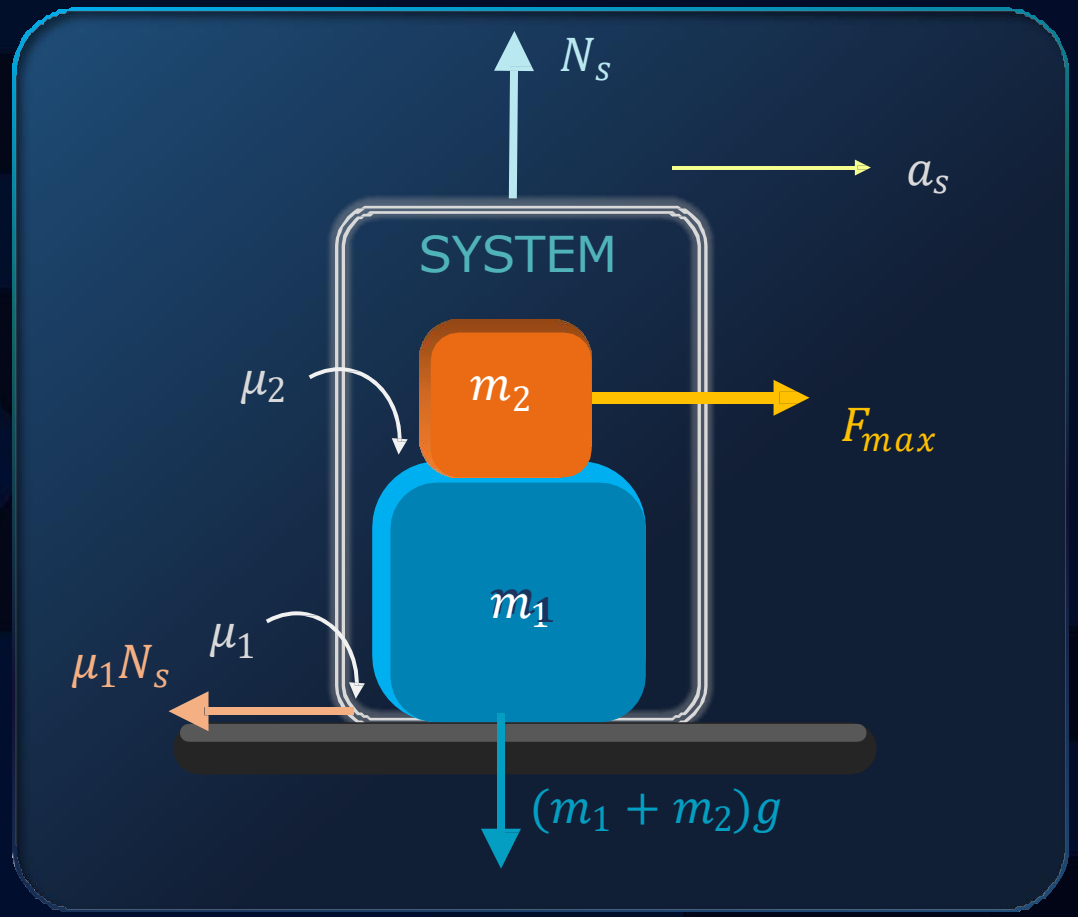
$$a_s = \frac{(\mu_2 - \mu_1) m_2}{m_1} g - \mu_1 g$$





Force applied on the Top block

What is the maximum force applied on the block m_2, so that both the blocks move together with a common acceleration?



$$F_{max} - \mu_1 N_s = Ma_s$$

$$F_{max} = Ma_s + \mu_1 N_s$$

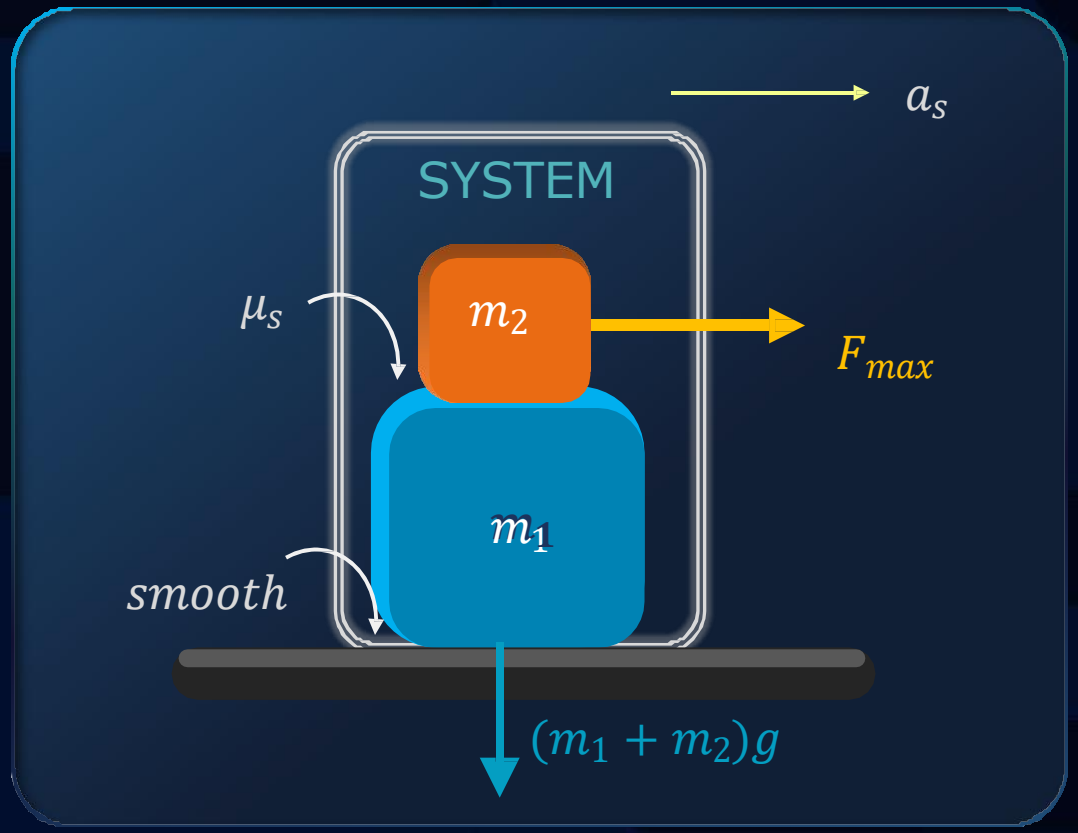
$$F_{max} = (m_1 + m_2)\frac{(\mu_2 - \mu_1)m_2}{m_1}g - \mu_1 g + \mu_1(m_1 + m_2)g$$

$$F_{max} = (\mu_2 - \mu_1)\frac{m_2}{m_1}(m_1 + m_2)g$$

Force applied on the Top block

What is the maximum force applied on the block m_2, so that both the blocks move together with a common acceleration?

Case : When the ground is smooth $\Rightarrow \mu_1 = 0, \mu_2 = \mu_s$



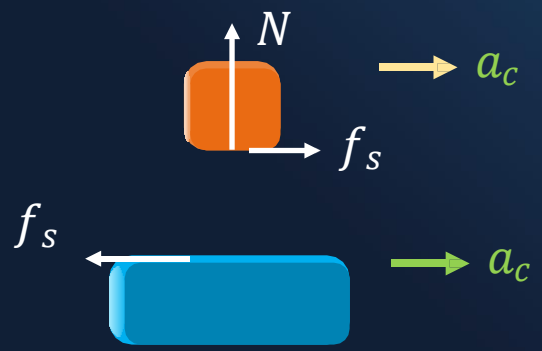
$$F_{max} = (\mu_2 - \mu_1)\frac{m_2}{m_1}(m_1 + m_2)g$$

$$F_{max} = \mu_s \frac{m_2}{m_1}(m_1 + m_2)g$$

$F \leq F_{max}$

YES

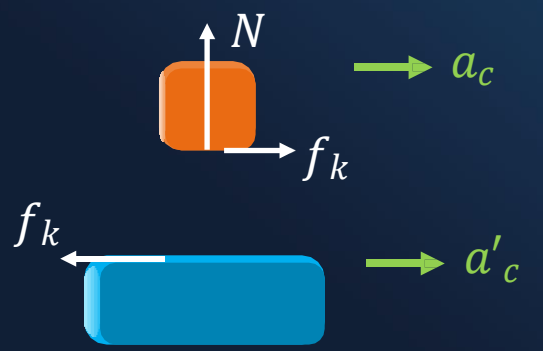
Blocks will move together with common acceleration



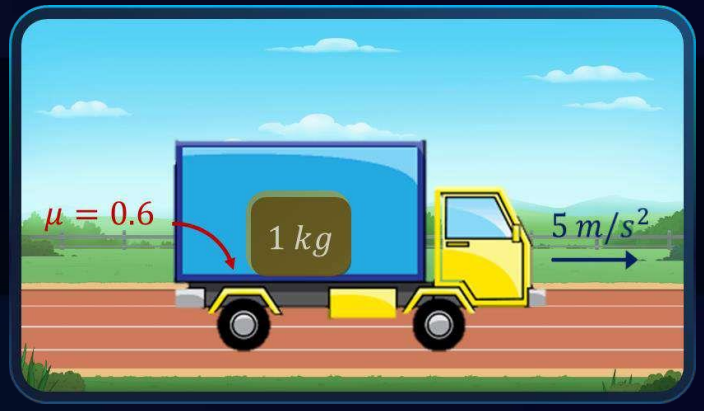
NO

There is relative motion between the blocks

⇒ Find acceleration of blocks from their respective FBDs



On the horizontal surface of a truck a block of mass $1\ kg$ is placed $(\mu = 0.6)$ and truck is moving with acceleration $5\ m/s^2$, then the frictional force on the block will be:



Given: $m = 1\ kg, \mu = 0.6, a = 5\ m/s^2$

To find: Frictional force on the block, f

Solution: Limiting frictional force, $f_l = \mu N = \mu mg$

$f_l = (0.6)(1)(10) = 6\ N$ .....(1)

Block is in non-inertial frame:

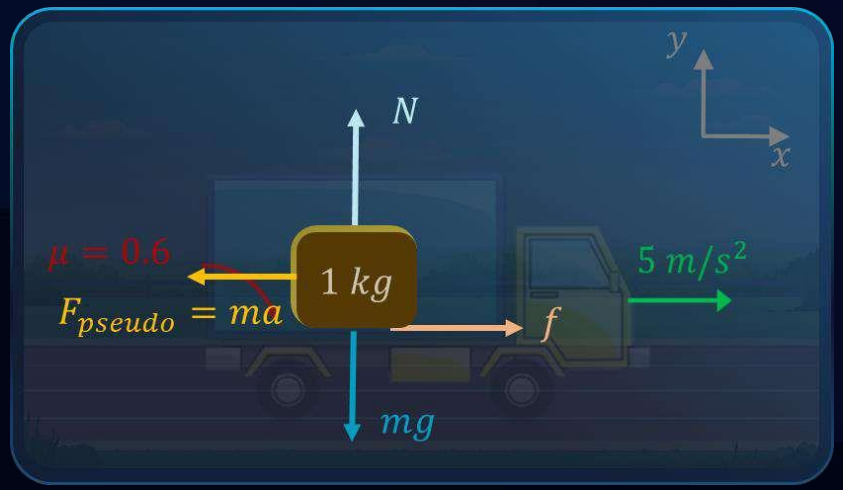
Pseudo force $= ma$

$F_{pseudo} = (1)(5)$

$F_{pseudo} = 5\ N$ .....(2)

From (1) and (2), $F_{pseudo} < f_l$

$f = 5\ N$



A heavy uniform chain lies on a horizontal tabletop. If the coefficient of friction between the chain and the table surface is 0.25, then the maximum fraction of the length of the chain that can hang over one edge of the table is

Solution: As it is a uniform chain, $\frac{M}{L} = \frac{m_x}{l_x} = \frac{m_y}{l_y}$

$$m_x = \frac{M}{L}x \qquad m_y = \frac{M}{L}(L - x)$$



For the chain to hang - Gravitational pull due to hanging part is balanced by frictional force on chain.

1 $W_x = f$

$m_x g = \mu N$

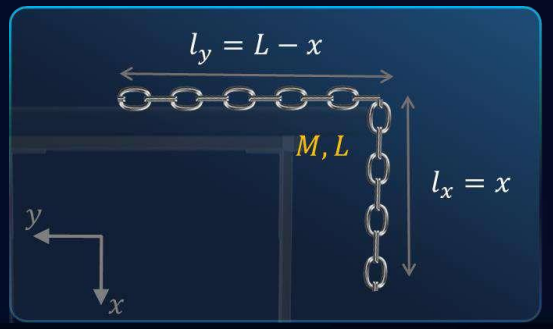
$\frac{M}{L}xg = \mu\frac{M}{L}(L - x)g$

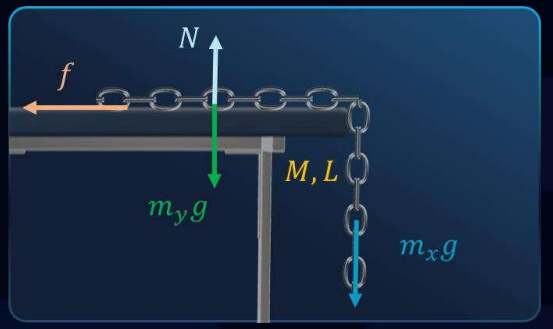
$gx = \mu\,(L - x)g$

2 $\frac{x}{L} = \frac{\mu}{1 + \mu}$

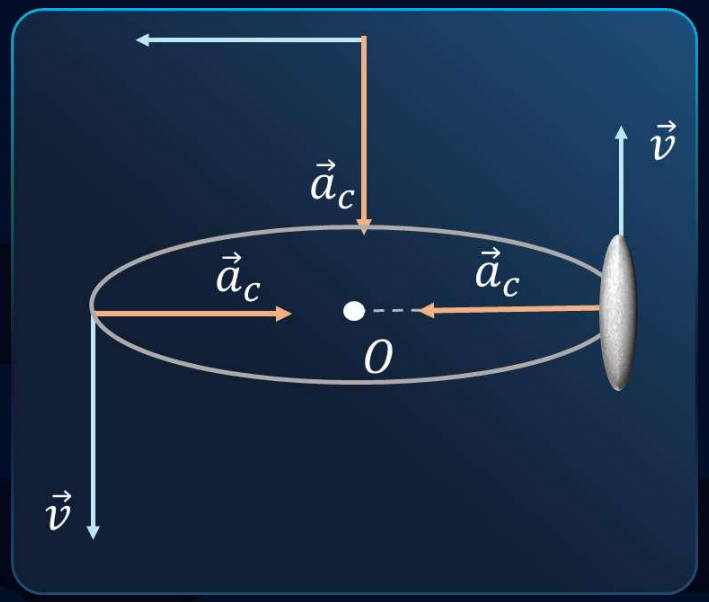
$\frac{x}{L} = \frac{0.25}{1 + 0.25} \times 100$

$\frac{x}{L} = 20\%$



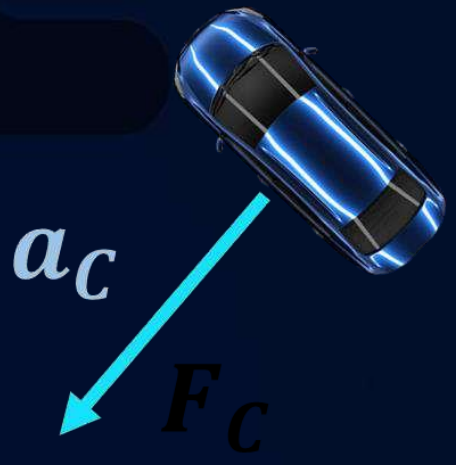


Centripetal acceleration



$$a_c = \frac{v^2}{R} = \omega^2 R$$

Centripetal force is a force on an object directed towards the center of a circular path that keeps the object on the path.



Centripetal Force:

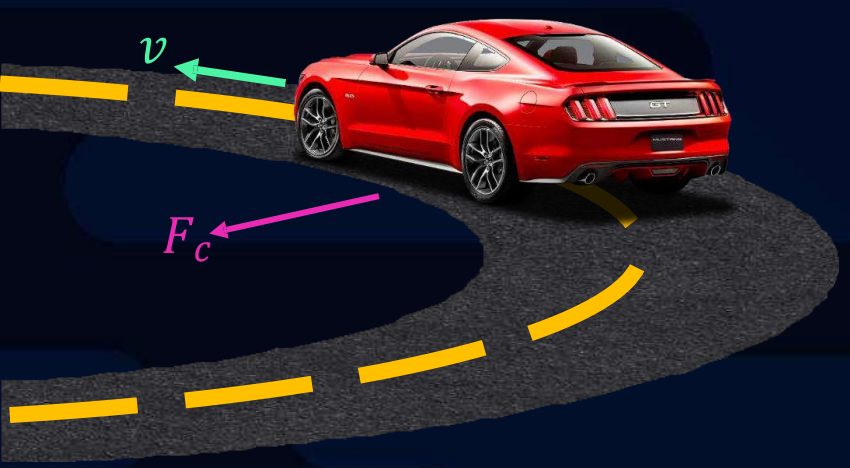
$$\Sigma \vec{F}_c = m\vec{a}_c$$

$$\Sigma |\vec{F}_c| = m|\vec{a}_c|$$

$$\Sigma F_c = \frac{mv^2}{R} = m\omega^2 R$$

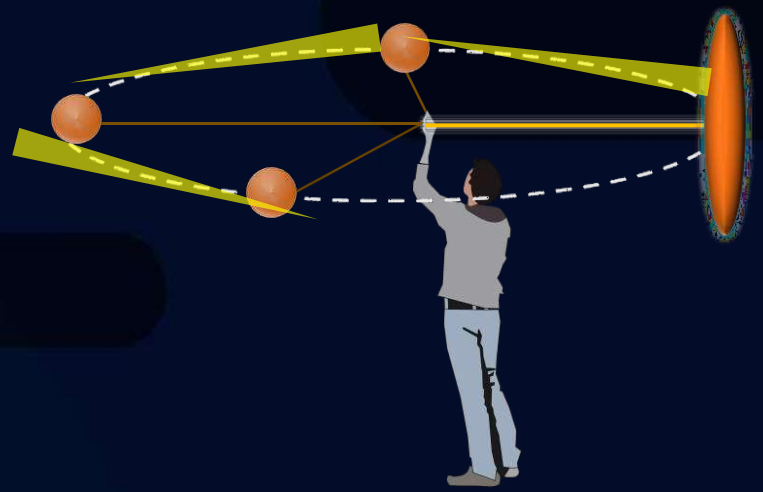
Examples of Centripetal Force

Friction Force:



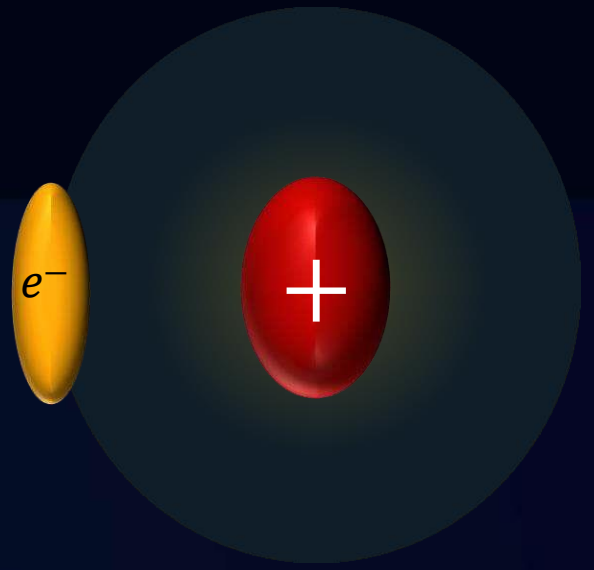
$$F_c = \frac{mv^2}{R} = m\omega^2 R$$

Tension in string:



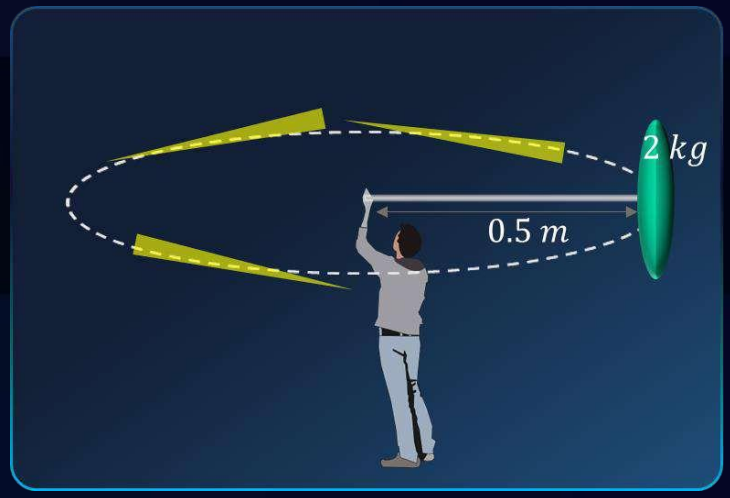
$$T = \frac{mv^2}{R} = m\omega^2 R$$

Electrostatic Force:



$$F_e = \frac{mv^2}{R} = m\omega^2 R$$

A stone of mass $2\ kg$ is tied to a string of length $0.5\ m$. If the breaking tension of the string is $900\ N$, then the maximum angular velocity the stone can have in uniform circular motion is



Given: $m = 2\ kg, r = 0.5\ m, T = 900\ N$

To find: ω_{max}

Solution: The necessary centripetal force is provided by the Tension force

$$F_c = mr\omega^2$$

$$T = m \times r \times \omega^2$$

$$\omega = \sqrt{\frac{T}{m \times r}} = \sqrt{\frac{900}{2 \times 0.5}}$$

$$\omega = 30\ rad/s$$

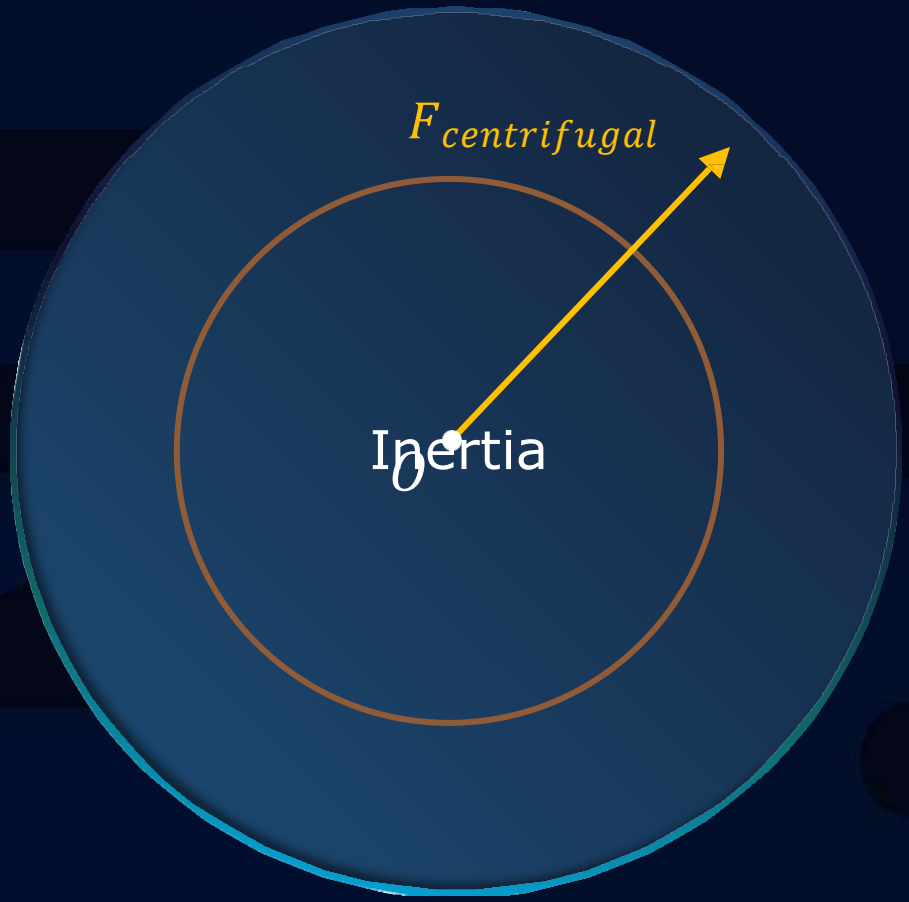
a. $30\ rad/s$

b. $20\ rad/s$

c. $10\ rad/s$

d. $25\ rad/s$

Centrifugal force



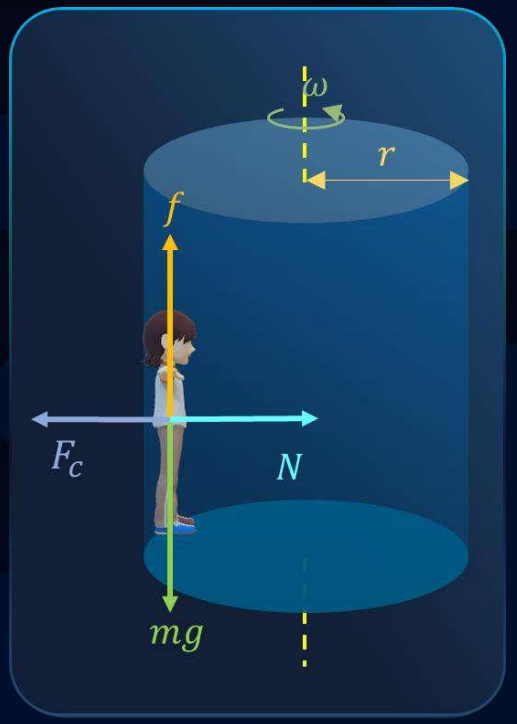
It is a fictitious or pseudo force.

$\not{F}_{centrifugal} = -\not{F}_{centripetal}$

It is considered in non-inertial frame of reference.

If the radius of the rotor is r and the coefficient of static friction between the wall and the person is μ, find the minimum speed at which the floor may be removed. (Non- Inertial frame of reference)

Man's frame (Non – inertial)



Along the radial direction

$$F_c - N = 0 \Rightarrow F_c = N$$

$$\frac{mv^2}{r} = N \qquad \text{.....(1)}$$

Along the vertical direction

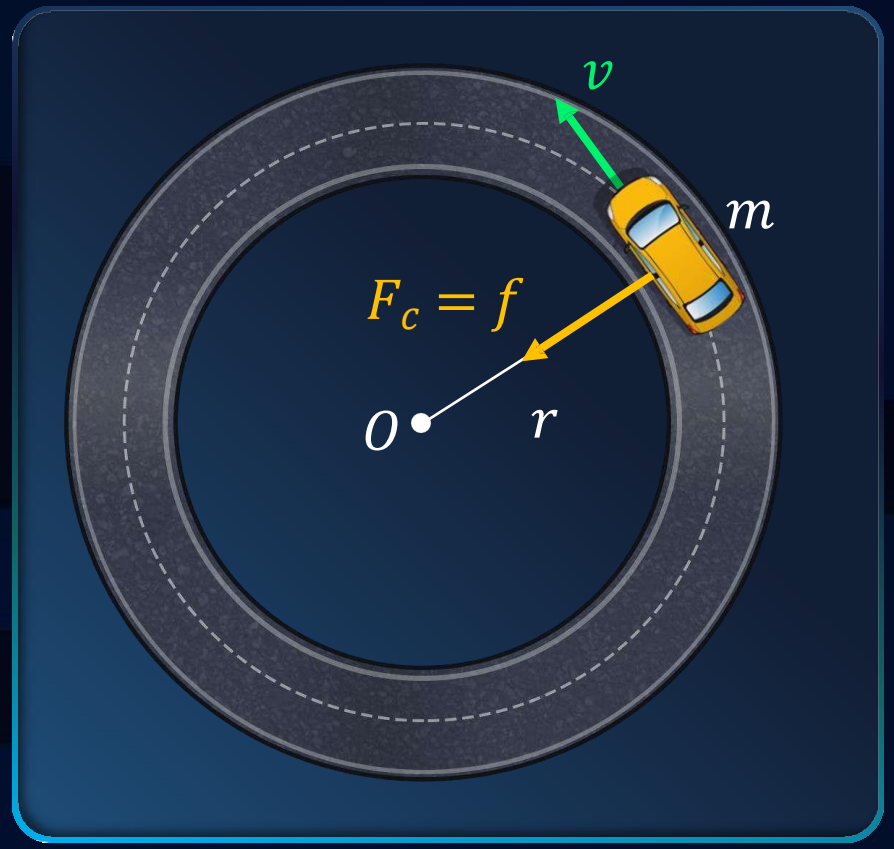
$$f = \mu N = mg \qquad \text{.....(2)}$$

From equation (1) and (2)

$$\frac{\mu m v^2}{r} = mg \Rightarrow \boxed{v = \sqrt{\frac{rg}{\mu}}}$$



Horizontal circular track with friction



Friction provides the Centripetal Force

$$f = F_c = \frac{mv^2}{r}$$

Limiting frictional force, $f_l = \mu N = \mu mg$

For a safe turn without sliding $f \leq f_l$

$$\frac{mv^2}{r} \leq f_l$$

$$\frac{mv^2}{r} \leq \mu mg$$

$$v \leq \sqrt{\mu r g}$$

If v ↑, the car skids outwards.

If v ↓, friction adjusts itself to prevent skidding.

A car of mass $800\ kg$ moves on a circular track of radius $40\ m$. If the coefficient of friction is 0.5, then the maximum speed with which the car can move is (Take $g = 9.8\ m/s^2$)

Given : $m = 800\ kg, r = 40\ m, \mu = 0.5$

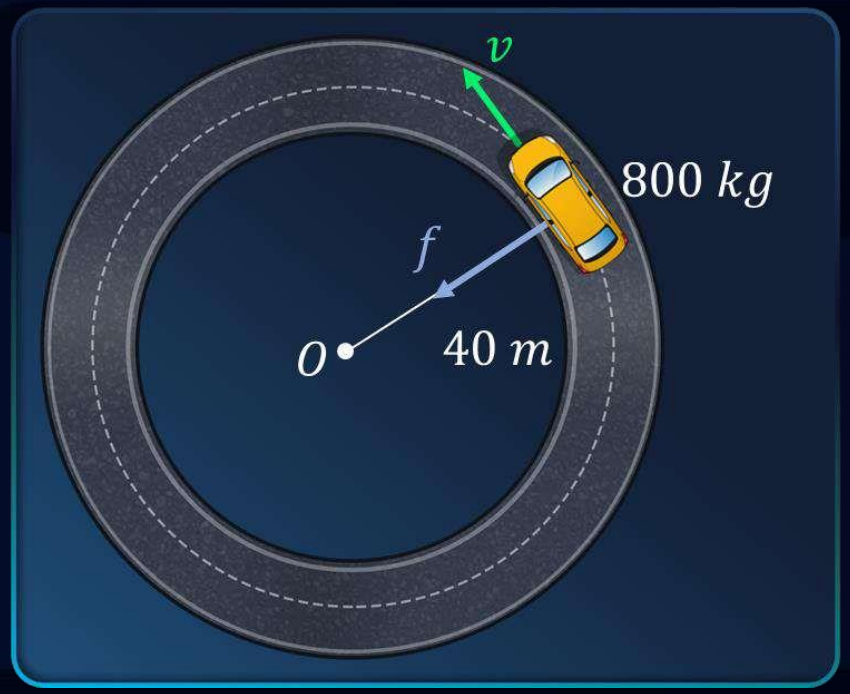
To Find : Maximum speed (v_{max})

Solution : Condition for maximum speed

$$v \leq \sqrt{\mu r g}$$

$$v_{max} = \sqrt{\mu r g}$$

$$v_{max} = \sqrt{0.5 \times 40 \times 10}$$



$v_{max} \approx 14\ m/s$

a. $7\ m/s$

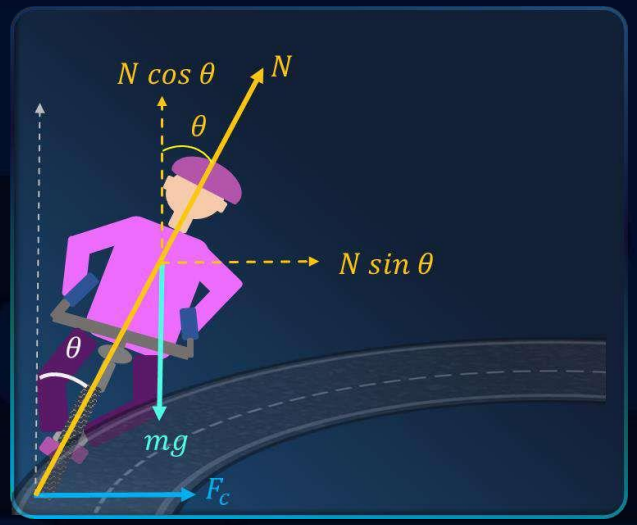
b. $8\ m/s$

c. $14\ m/s$

d. $12\ m/s$

Bending of cyclist (without friction)

Necessary centripetal force is provided by a component of Normal reaction.



Along radial direction

$$N \sin\theta = \frac{mv^2}{r}$$

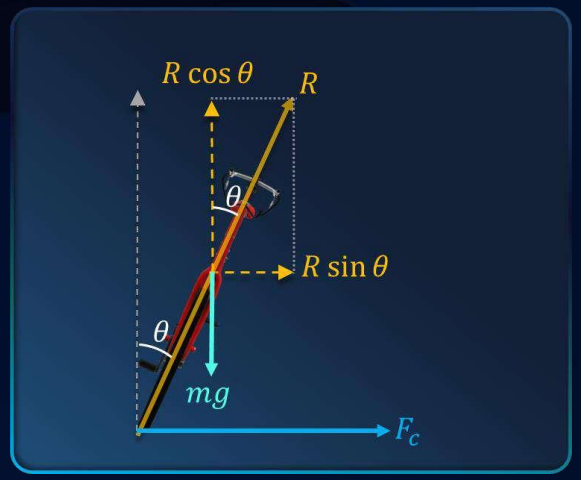
$$\frac{N \sin\theta}{N \cos\theta} = \frac{\frac{mv^2}{r}}{mg}$$

$$\tan\theta = \frac{v^2}{rg}$$

Along vertical direction

$$N \cos\theta = mg$$

Net force (Friction + Normal force) = R



Along radial direction

$$R \sin\theta = \frac{mv^2}{r}$$

$$\frac{R \sin\theta}{R \cos\theta} = \frac{\frac{mv^2}{r}}{mg}$$

$$\tan\theta = \frac{v^2}{rg}$$

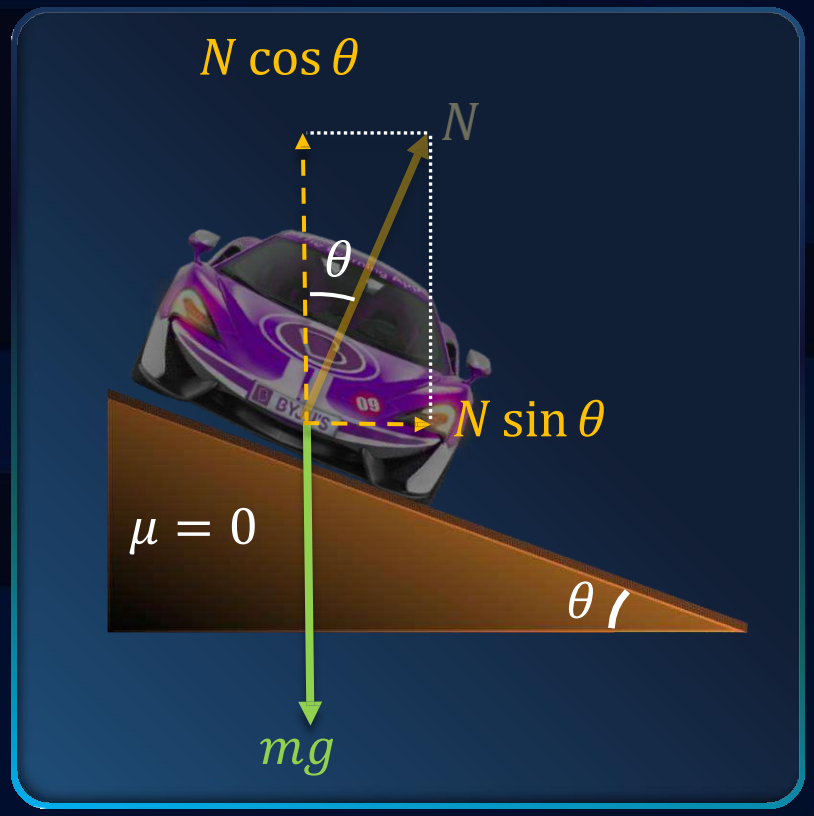
Along vertical direction

$$R \cos\theta = mg$$

$$v = \sqrt{rg \tan\theta}$$

Banking of roads

The outer edge of the road is raised above the inner edge making an inclination(θ) with the horizontal.



Case 1: Without Friction

Applying Newton's law,

$$N\sin\theta = \frac{mv^2}{r}$$

$$N\cos\theta = mg$$

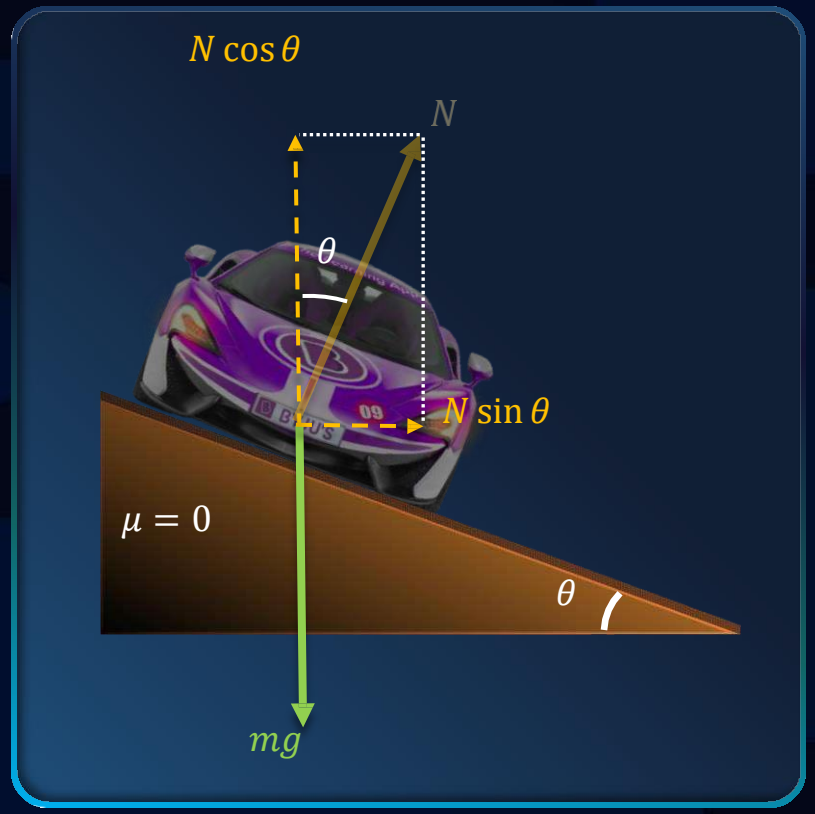
$$\tan\theta = \frac{v^2}{rg}$$

$$v_{max} = \sqrt{rg\tan\theta}$$

- If v ↑, the car skids outwards.
- If v ↓, the car skids inwards.

If a car is to travel with a speed v along the frictionless banked circular track of radius r, the required angle of banking so that the car does not skid is

Solution :



Solution:

Along radial direction

$$N\sin\theta = \frac{mv^2}{R}$$

$$\frac{N\sin\theta}{N\cos\theta} = \frac{v^2}{rg}$$

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

Along vertical direction

$$N\cos\theta = mg$$

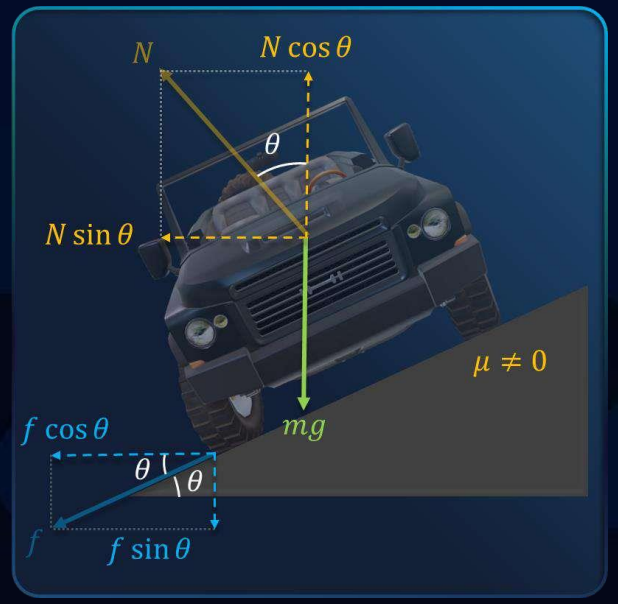
a. $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

c. $\theta = \tan^{-1}\left(\frac{r^2}{vg}\right)$

b. $\theta = \tan^{-1}\left(\frac{v}{rg}\right)$

d. $\theta < \tan^{-1}\left(\frac{v^2}{rg}\right)$

Banking of Roads



Case 2: with Friction

- Speed $v > \sqrt{rg \tan\theta}$

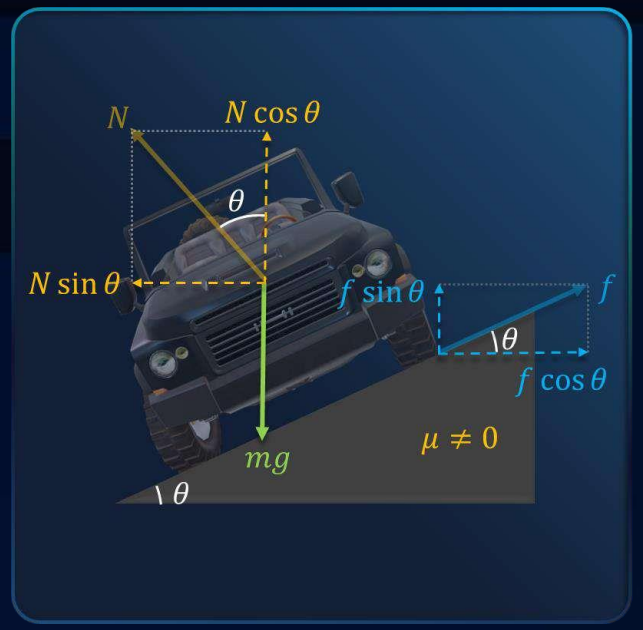
$$N \sin\theta + f \cos\theta = \frac{mv^2}{r} \quad \ldots\ldots\ldots(1)$$

$$N \cos\theta - f \sin\theta = mg \quad \ldots\ldots\ldots(2)$$

From (1) and (2)

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{rg}$$

$$v_{max} = \sqrt{\frac{rg(\tan\theta + \mu)}{(1 - \mu\tan\theta)}}$$



Case 2: with Friction

- Speed $v > \sqrt{rg \tan\theta}$

$$N \sin\theta - f \cos\theta = \frac{mv^2}{r} \quad \ldots\ldots\ldots(1)$$

$$N \cos\theta + f \sin\theta = mg \quad \ldots\ldots\ldots(2)$$

From (1) and (2)

$$\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta} = \frac{v^2}{rg}$$

$$v_{min} = \sqrt{\frac{rg(\tan\theta - \mu)}{(1 + \mu\tan\theta)}}$$

Safe speed

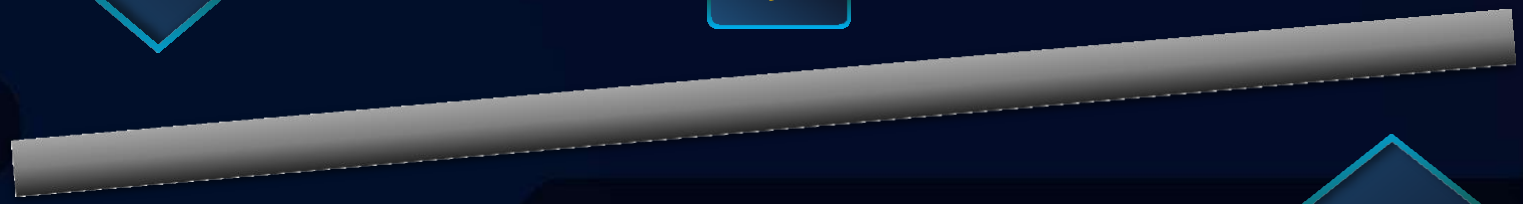
To avoid skidding, safe speed of the vehicle should be in the range,

$$v_{min} \leq v \leq v_{max}$$

$v < v_{min}$

Car skids radially outwards

v



Car skids radially inwards

$v > v_{max}$

Key Takeaways

$v_{required}$

Only Friction

$\mu \neq 0 \quad \theta = 0$

$v = \sqrt{\mu r g}$

Only Banking

$\mu = 0 \quad \theta \neq 0$

$v = \sqrt{rg \tan\theta}$

Banking with friction

$\mu \neq 0 \quad \theta \neq 0$

$$\sqrt{\frac{rg(\tan\theta - \mu)}{(1 + \mu\tan\theta)}} < v < \sqrt{\frac{rg(\tan\theta + \mu)}{(1 - \mu\tan\theta)}}$$

μ is coefficient of friction.

θ is the Angle of banking.

A car is negotiating a curved road of radius $30\ m$. The road is banked at an angle $45°$. The coefficient of friction between the tyres of the car and the road is 0.5. The maximum safe velocity on this road is

Given : $r = 30\ m,\ \theta = 45°,\ \mu = 0.5$

To Find : Maximum speed (v_{max})

Solution : $v_{max} = \sqrt{\frac{rg(\tan\theta + \mu_s)}{(1 - \mu_s \tan\theta)}}$

$$v_{max} = \sqrt{\frac{30 \times 10(\tan 45° + 0.5)}{(1 - (0.5 \times \tan 45°))}}$$

$v_{max} = 30\ m/s$

a. $10\ m/s$

b. $30\sqrt{10}\ m/s$

c. $30\ m/s$

d. $10\sqrt{10}\ m/s$